

Index

- Abel, N., 240
Abel's integral equation, 242
Absolutely integrable, 39, 85
Addition formula,
 Bessel functions, 279
Airy stress function, 152–154
Analytic function, 337
Andrews, L. C., 6, 38, 103, 108, 136, 226,
 231, 298, 304
Applications,
 discrete transforms, 330–333
 double Fourier transform, 121
 finite transforms, 294–296, 301, 302, 306
 Fourier transforms, 102–161
 Hankel transform, 285–290
 Laplace transform, 218–244
 Mellin transform, 266–272
Archimedes, 37
Asymptotic formula,
 Bessel functions, 281
 complementary error function, 19–20,
 214
Beam theory, 106
Bernoulli, D., 23, 37, 127, 274
Bessel, F. W., 22
Bessel functions, 21–29, 60, 66–67, 74–
 76, 78, 84–85, 101, 149, 155, 159,
 161, 185–189, 197–198, 200, 209, 220,
 223, 239, 244, 250–251, 269–270, 274–
 290
 addition formula for, 279
 asymptotic formula for, 281
 differential equation for, 27, 281, 303,
 307
 graph of, 22, 26
 Jacobi-Anger expansion, 27
 modified, 25–29, 159
 of the first kind, 21–25
 of the second kind, 25
 properties of, 24–25
 series representation of, 21
Bessel equation, 27, 281, 303, 307
 eigenfunctions of, 303
 eigenvalues of, 303
Beta function, 15
Bibliography, 335–336
Biharmonic equation, 142, 147–149, 152
 elasticity, 152–154
 viscous fluid, 147–149
Bilateral Laplace transform, 214–216
Binomial coefficient, 14
Boundary value problems, 103–112, 131–
 138
 mixed, 137–138
 of the first kind, 132
 of the second kind, 132
 singular, 103
Branch point, 338
Cauchy, A. L., 38
Cauchy integral theorem, 338
Cauchy–Riemann equations, 149, 337
Causal function, 91, 163

- Characteristic function, 4–5, 156–161
 Churchill, R. V., 168, 202, 293
 Complementary error function, 17, 19–20,
 120, 122, 124, 169, 171, 180, 183–184,
 187–188, 210, 214, 216, 220, 224, 230,
 233–234, 236, 261
 asymptotic series for, 19–20, 214
 Complex variables, 337–339
 analytic function, 337
 branch point, 338
 Cauchy's integral theorem, 338
 Cauchy–Riemann equations, 337
 poles, 338
 residue theorem, 338–339
 Convolution formula for
 cosine transform, 82, 138
 discrete Fourier transform, 315
 double Fourier transform, 99–100
 Fourier transform, 78–85, 115, 117, 129,
 137, 149
 Laplace transform, 194–198, 225, 230
 Mellin transform, 249, 266
 sine transform, 82
 Z transform, 327
 Cooley, J. W., 316
 Cosine integral, 180, 261
 Cosine integral representation, 43–46
 Cosine transform, 51–53, 62–63, 67–68,
 118–119, 247, 250
 inverse, 51
 D'Alembert, J. 127
 D'Alembert solution, 127
 Delta function. *See* Impulse function
 Delta sequence, 36
 Derivative properties of
 cosine transform, 62–63
 double Fourier transform, 99
 finite Fourier transforms, 293–294
 finite Hankel transform, 304–305
 Fourier transform, 61–64
 generalized finite Fourier transform, 301
 Hankel transform, 280–281
 Laplace transform, 173–177
 Mellin transform, 249
 Difference, 330
 Difference equations, 330–333
 Diffusion equation. *See* Heat equation
 Dirac delta function. *See* Impulse function
 Dirac, P. A. M., 33
 Dirichlet condition, 114, 132
 Dirichlet problem, 132
 axisymmetric, 285–286
 half-plane, 133–135
 infinite strip, 136–137
 Dirichlet series, 252
 Discrete transforms, 310–334
 applications, 330–333
 fast Fourier, 316–319
 Fourier, 311–319
 inverse, 313, 328
 properties of, 315–316, 324–325
 sampling continuous functions, 311
 table of, 334
 Z, 321–334
 Doetsch, G., 292
 Double Fourier transform, 98–101, 121,
 274–275
 Dual integral equations, 287
 Duhamel's principle, 122–123
 Duplication formula, 13, 15, 66
 Eigenfunctions, 108, 298–299
 Eigenvalues, 108, 298–299
 Elasticity, 151–155
 Airy stress function, 152
 compatibility condition, 152
 equations of motion, 151
 equilibrium, 152–154
 stress components, 151, 153–154
 Elliptic integral, 83
 Energy spectrum, 79
 Entire function, 16
 Erdelyi, A., 98
 Error function, 16–21, 124, 182–184, 187–
 188, 200, 210, 214, 216, 220, 224,
 233–234, 236
 complementary. *See* complementary er-
 ror function
 graph of, 17
 properties of, 16–17
 Euler formulas, 49, 126, 233
 Euler, L., 7, 23, 106, 151
 Exponential integral, 180, 216, 261
 Exponential order, 166
 Fast Fourier transforms, 316–319
 Fibonacci sequence, 333
 Final value theorem,
 Laplace transform, 212
 Z transform, 329
 Finite transforms, 291–309
 applications, 294–296, 306
 cosine, 294
 Fourier, 291–298
 generalized Fourier, 300–301
 inverse, 292, 294, 300, 302, 304
 Hankel, 303–309
 sine, 292–293
 Sturm-Liouville, 298–309
 Fluid flow. *See* Hydrodynamics
 Fourier integral representations, 38–49,
 106–107

- cosine, 43–46
- sine, 43–46
- Fourier integral theorem, 40
 - exponential form of, 50
 - proof of, 47–49
 - Riemann-Lebesgue lemma, 47
- Fourier, J., 37, 113
- Fourier series, 38, 299
 - cosine, 292
 - generalized, 299
 - sine, 292
- Fourier's law, 113
- Fourier transform, 2, 49–161, 208
 - applications, 102–161
 - convolution integral, 78–85, 99–100, 115, 117, 129, 137–138, 149
 - cosine, 51–53
 - definition of, 2, 50
 - discrete, 311–334
 - fast, 316–319
 - inverse, 50, 59
 - method of stationary phase, 97–98
 - multiple, 98–101, 121, 274–275
 - properties of, 58–65
 - residue methods, 67–74
 - sine, 51–53
 - table of, 340–343
- Fredholm integral equation, 108
- Fresnel integrals, 17–21, 58, 131, 189, 234
 - graph of, 18
 - series representation of, 20
- Gamma function, 7–16, 66, 166, 170, 247, 250–251, 256–259, 264
 - argument negative, 8
 - graph of, 9
 - poles of, 8
 - properties of, 13
- Gaussian random variable, 20, 157
- Generalized finite Fourier transform, 300–301
- Generalized functions, 85–91
 - See also* Impulse function
- Goodier, J. N., 152
- Gradshteyn, I. S., 159
- Green's function, 226
- Hankel transform, 3, 274–290
 - applications, 285–290
 - definition of, 3, 275
 - finite, 303–309
 - inverse, 275
 - properties of, 278–282
 - table of, 290
- Harmonic function, 132
- Harrington, W. J., 259
- Heat equation, 4, 113–125, 229–231
 - axisymmetric, 288
 - finite interval, 230–231, 295–296, 301–302
 - infinite line, 114–117
 - infinite rectangle, 121
 - semiinfinite line, 118–120, 229–230
- Heaviside expansion theorem, 205
- Heaviside, O., 162
- Heaviside unit function, 29–31, 41, 66, 88–89, 104, 172–173, 222, 232, 236, 276, 328
- Helstrom, C. W., 157
- Hilbert-Schmidt kernel, 108
- Hilbert transform, 91–97
- Hydrodynamics, 141–151
 - continuity equation, 141
 - equation of motion, 141
 - ideal fluid, 141
 - incompressible flow, 141
 - irrotational flow, 143–144
 - sources and sinks, 141
 - steady flow, 142, 147–149
 - stream function, 142
 - surface waves, 144–147
 - two-dimensional flow, 141
 - viscous fluid, 141
 - vorticity, 141
- Impulse function, 31–36, 85–91, 122, 150, 189, 226, 232, 236, 311, 321–322
 - definition of, 32
 - sifting property of, 33
- Impulse response function, 224–227
- Influence function, 196
- Initial value problems, 221–228
- Initial value theorem,
 - Laplace transform, 211
 - Z transform, 329
- Integral equations, 4–5, 107–112, 238–244
 - convolution type, 108, 238–244
 - dual, 287
 - Fredholm, 108
 - Hilbert-Schmidt kernel, 108
 - of the first kind, 107, 238–239
 - of the second kind, 108, 240
 - Volterra type, 108, 238–244
- Inversion formula, 3
 - cosine transform, 51
 - discrete Fourier transform, 313
 - double Fourier transform, 98
 - finite cosine transform, 294
 - finite Hankel transform, 304
 - finite sine transform, 292
 - Fourier transform, 50, 59
 - generalized finite Fourier transform, 301
 - Hankel transform, 275
 - Laplace transform, 164–201

- Mellin transform, 246
 sine transform, 51
 Z transform, 328
- Jacobi–Anger expansion, 27
- Kellogg, O. D., 132
- Kernel, 2
 Hilbert–Schmidt, 108
- Kramers–Krönig relations, 94
- Laguerre polynomials, 181, 189
- Laplacian, 113
- Laplace, P. S., 38, 162
- Laplace's equation. *See* Potential equation
- Laplace transform, 2, 162–244
 applications, 218–244
 convolution integral, 194–198, 225, 230
 definition of, 2, 164
 existence theorem for, 167
 inverse, 164, 190–200
 of periodic functions, 177–179
 properties of, 170–181, 211–214
 table of, 344–347
 two-sided, 214–216
- Legendre, A. M., 7
- Legendre polynomials, 75
- Leibnitz's rule, 267
- LePage, W. R., 168, 216
- Lerche's theorem, 190
- Linear integral equations. *See* Integral equations
- Linearity property of
 discrete Fourier transform, 320
 Fourier transform, 59
 Hankel transform, 278
 Laplace transform, 170, 191
 Mellin transform, 248
 Z transform, 324
- Little, R. M., 151
- Lovitt, W. L., 108
- MacFarlane, G. G., 263
- Maclaurin, C., 37
- Mellin transform, 3, 245–273
 applications, 262–272
 convolution integral, 249, 266
 definition of, 3, 246
 inverse, 246, 256–259, 263
 polar coordinates, 259–260
 properties of, 248–251
 residue methods, 254–273
 table of, 273
- Miller, K. S., 332
- Mixed boundary value problem, 286
- Modified Bessel functions, 25–29, 159
 graph of, 26
 See also Bessel functions
- Multiple Fourier transform. *See* Double Fourier transform
- Neumann condition, 114, 132
- Neumann problem, 132, 135–136, 144
 half-plane, 135–136, 144
- Normal random variable, 20, 157
- Null function, 190
- One-sided Green's function, 226
- Operational properties of
 discrete Fourier transform, 315–316, 324–325
 Fourier transform, 58–65
 Hankel transform, 278–282
 Laplace transform, 170–181, 211–213
 Mellin transform, 248–251
 Z transform, 324–325
- Orthogonality relation for
 Hilbert transform, 97
 Sturm-Liouville problem, 299
- Papoulis, A., 157
- Parseval's relation for
 cosine transform, 84
 discrete Fourier transform, 315
 Fourier transform, 79
 Hankel transform, 279
 Hilbert transform, 97
 sine transform, 84
- Partial fractions, 192–194, 325–326
- Periodic functions, 177–179
- Periodic sequences, 314
- Piecewise smooth, 39
- Poisson integral formula, 134
- Poisson, S. D., 162
- Poles, 338
- Potential equation, 131–138, 143–144
 axisymmetric, 285–286
 half-plane, 133–136, 143–144
 infinite strip, 136–137
 rectangle, 294–295
 semiinfinite strip, 137–138
- Potential function, 132, 143
- Principal value, 2
- Probability and statistics, 4–5, 156–161, 265–267
 characteristic functions, 4–5, 156–161
 density functions, 156, 159–161, 265–267
 distribution function, 156
 statistical moments, 156
- Rectangle function, 30
- Residue methods,
 Fourier transform, 67–74
 Laplace transform, 200–210

- Mellin transform, 254–262
- Z transform, 327–328
- Residue theorem, 338–339
- Riemann, G., 263
- Riemann-Lebesgue lemma, 47
- Riemann zeta function. *See* Zeta function
- Robin's condition, 114
- Ryzhik, I. M., 159
- Sampled function, 311–312, 321
- Scaling property of
 - Fourier transform, 60
 - Hankel transform, 278
 - Laplace transform, 171
 - Mellin transform, 248
- Schwartz, L., 85
- Self-reciprocal function, 56, 278, 284
- Shift properties of
 - discrete Fourier transform, 315
 - double Fourier transform, 99
 - Fourier transform, 60–61
 - Laplace transform, 171–173, 191
 - Mellin transform, 248
 - Z transform, 324–325
- Shivamoggi, B. K., 141
- Signum function, 34, 52, 88, 90–91, 110, 148–149
- Sine integral, 43, 177, 216
- Sine integral representation, 43–46
- Sine transform, 51–53, 63, 119–120, 208
 - inverse, 51
- Singularity,
 - branch point, 338
 - isolated, 338
 - pole, 338
- Sneddon, I. N., 213, 275, 276, 287, 306
- Special functions, 6–36
 - beta function, 15
 - Bessel functions, 21–29
 - cosine integral, 180, 261
 - elliptic integral, 83
 - error functions, 16–21
 - Fresnel integrals, 17–21, 58, 131, 189, 234
 - gamma function, 7–16
 - Heaviside unit function, 29–31
 - impulse function, 31–36
 - Laguerre polynomials, 181
 - Legendre polynomials, 175
 - rectangle function, 30
- signum function, 34
- sine integral, 43, 177, 216
- Spectrum, 86
- Springer, M. D., 265
- Stationary phase method, 97–101, 146–147
- Statistical moments, 156
- Stream function, 142
- Sturm-Liouville problem, 298, 300, 303
- Sturm-Liouville transform, 298–309
 - applications, 301–302, 306
 - finite Hankel, 303–309
 - generalized finite Fourier, 300–301
 - inverse, 300, 304
- Tautochrone problem, 240–242
- Taylor, B., 37
- Thompson, W. E., 265
- Timoshenko, S., 127, 152
- Titchmarsh, E. C., 40
- Translation property of
 - Laplace transform, 172, 232
 - Mellin transform, 248
- Tukey, J. W., 316
- Two-sided Laplace transform, 214–216
- Unit step function. *See* Heaviside unit function
- Vibrating
 - beam, 127–129, 233–234
 - membrane, 287–288
 - string, 125–127, 231–232
- Volterra integral equation, 108, 238, 241
- Watson, G. N., 264
- Watson's lemma, 213–214
- Wave equation, 4, 125–131, 231–234
 - infinite line, 125–127
 - semiinfinite line, 231–232
- Whittaker, E. T., 264
- Widder, D. V., 216
- Zemanian, A. H., 85
- Zeta function, 253, 263–264, 268–269
 - properties of, 264
- Z transform, 321–334
 - applications, 330–334
 - inverse, 325–328
 - properties of, 324–325, 329
 - table of, 334