Hands-on Morphological Image Processing

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Edward R. Dougherty • Roberto A. Lotufo

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Introduction to the Series

Since its conception in 1989, the Tutorial Texts series has grown to more than 60 titles covering many diverse fields of science and engineering. When the series was started, the goal of the series was to provide a way to make the material presented in SPIE short courses available to those who could not attend, and to provide a reference text for those who could. Many of the texts in this series are generated from notes that were presented during these short courses. But as stand-alone documents, short course notes do not generally serve the student or reader well. Short course notes typically are developed on the assumption that supporting material will be presented verbally to complement the notes, which are generally written in summary form to highlight key technical topics and therefore are not intended as stand-alone documents. Additionally, the figures, tables, and other graphically formatted information accompanying the notes require the further explanation given during the instructor's lecture. Thus, by adding the appropriate detail presented during the lecture, the course material can be read and used independently in a tutorial fashion.

What separates the books in this series from other technical monographs and textbooks is the way in which the material is presented. To keep in line with the tutorial nature of the series, many of the topics presented in these texts are followed by detailed examples that further explain the concepts presented. Many pictures and illustrations are included with each text and, where appropriate, tabular reference data are also included.

The topics within the series have grown from the initial areas of geometrical optics, optical detectors, and image processing to include the emerging fields of nanotechnology, biomedical optics, and micromachining. When a proposal for a text is received, each proposal is evaluated to determine the relevance of the proposed topic. This initial reviewing process has been very helpful to authors in identifying, early in the writing process, the need for additional material or other changes in approach that would serve to strengthen the text. Once a manuscript is completed, it is peer reviewed to ensure that chapters communicate accurately the essential ingredients of the processes and technologies under discussion.

It is my goal to maintain the style and quality of books in the series, and to further expand the topic areas to include new emerging fields as they become of interest to our reading audience.

Arthur R. Weeks, Jr.
University of Central Florida

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Preface

Morphological image processing has become a standard part of the imaging scientist's toolbox and today is applied daily to a wide range of industrial applications, including (and certainly not limited to) inspection, biomedical imaging, document processing, pattern recognition, metallurgy, microscopy, and robot vision. Because the morphological operations can serve as a universal language for image processing, their application is only limited by the ability to design effective algorithms and efficient computational implementation.

In the last decade, since the publication of *An Introduction to Morphological Image Processing* by SPIE Press, there have been many developments in morphological imaging, both in theory and practice. This book concentrates on applications. In keeping with the outlook of the previous book, we aim to provide a handbook that instructs how to analyze a problem and then how to develop successful algorithms based on the analysis. We take a holistic approach by showing how generic methods can be used in combination to solve practical problems. We include demonstrations to show how various morphological techniques can be combined to produce complete and effective algorithms.

In concentrating on applications we have not sacrificed careful definitions and explicit statement of operation properties. Indeed, skillful application requires that one understand the general filtering effects of an operation, and efficient implementation requires that one have knowledge of the operational simplifications available. Filter properties are integrated into the applications and their relevance is emphasized.

We have heard recently the comment that "morphology is an industrial subject." To a great extent this is true, although one should extend this to include research and development laboratories of all kinds. Several months ago, when asked to develop a high-throughput algorithm for analyzing genetic cell arrays, we suggested to our biological colleagues that we would use an algorithm known as the watershed. "Of course," they replied.

The book is *hands-on* in a very real sense. Most of the techniques used in the book are available in the **Morphology Toolbox**, and a great majority of the images shown in the text have been processed by the toolbox. These images, along with a demonstration version of the toolbox, are downloadable free from the web (`http://sourceforge.net/projects/pymorph`), so that the reader can actually process the images according to the examples and demonstrations in the text. There is a brief discussion in the first chapter as to how the toolbox correlates to the text, but we do not emphasize this relation throughout the text since we do not want to digress from the imaging essentials. Detailed use of the toolbox can be learned from the downloadable version. To assist the user, each chapter concludes

with a list of the toolbox operations used in the chapter, and the detailed demonstration sections include corresponding toolbox implementations. Each chapter also has its own bibliography.

We briefly describe the structure of the book. The first two chapters discuss the basic binary morphological operations, erosion and its dual dilation in the first chapter, and opening and its dual closing in the second. All of morphological image processing rests on these operations. Therefore, we treat them in great detail. The third and fourth chapters discuss the processing of binary images, the third focusing on applications of the primary operators, and the fourth on the hit-or-miss transform, which processes an image directly in terms of the foreground-background relation. We pay particular attention to morphological reconstruction in the third chapter because we believe that it is a very powerful tool for the development of algorithms based on inherent image structure.

Gray-scale morphological operators are discussed in the fifth chapter, and their application is treated in the following chapter. Once again we pay much attention to the role of reconstruction in applications. These tools have been more recently developed and we expect that they will lead to an ever-expanding range of application.

The seventh chapter is devoted to watershed-based segmentation. There are many variants of watershed segmentation. Our approach is to articulate the underlying principles while at the same time providing real-world applications. The key to successful segmentation is marker construction, and this issue is to some extent the focus of the chapter.

In his original (and highly mathematical) work, *Random Sets and Integral Geometry*, Georges Matheron, who along with Jean Serra founded the subject of mathematical morphology, comments, "Despite the purely mathematical nature of the present treatise, the formulation and the very choice of problems for solution are directly inspired by experimental techniques of texture analysis." It is not surprising, therefore, that morphological imaging is fundamental to a core understanding of texture. The sixth chapter discusses granulometric filters, which play a key role in describing and classifying texture and particle distributions. Whereas Matheron formalized their definition mathematically, granulometric-type methods are well known in sedimentology and the study of porous media.

The final chapter of the book concerns the automatic design of morphological operators. Morphological image processing is based on probing an image with structuring elements, and these determine the relationships within image structure that an algorithm can ascertain. Again quoting Matheron, "In general, the structure of an object is defined as the set of relationships existing between elements or parts of the object... Hence, this choice [of relationships]... determines the relative worth of the concept of structure at which we will arrive." In many cases it is possible to obtain satisfactory structuring elements by human ingenuity; however, when successful filtering requires hundreds or even thousands of structuring elements,

automatic design from training data becomes essential. It is here that morphological image processing meets computational learning.

Before closing this preface, we would like to acknowledge some of the people who helped make this book possible: Junior Barrera, Rubens C. Machado, Roberto Hirata, Jr., Nina S. T. Hirata, Marcel Brun, Yidong Chen, Seungchan Kim, Artyom Grigoryan, Ulisses Braga Neto, and the graduate students who attended the second semester-2002 Morphological Image Analysis course at University of Campinas, Brazil, testing a preliminary version of this book. We especially thank our wives Terry and Valéria for their ongoing support.

Finally, we hope you find this book both enjoyable and useful to your imaging work, in whatever your field.

Edward R. Dougherty
College Station, Texas, U.S.A.

Roberto A. Lotufo
Campinas, São Paulo, Brazil

List of Symbols

$A \cap B$	Intersection between two sets
$A \cup B$	Union between two sets
$f \wedge g$	Pixelwise minimum between two images
$f \vee g$	Pixelwise maximum between two images
$f \leq g$	Image f is beneath g
$f \geq g$	Image f is above g
f^c	Negation of image f
$f^{\dot{c}}$	Bounded negation of image f
$f - g$	Subtraction between two images
$f -^* g$	Bounded subtraction between two images
$f + g$	Addition between two images
$f +^* g$	Bounded addition between two images
$f \triangle g$	Symmetrical difference between f and g
$f \ominus g$	Erosion of f by g
$f \oplus g$	Dilation of f by g
$f \dot{\ominus} g$	Bounded erosion of f by g
$f \dot{\oplus} g$	Bounded dilation of f by g
$f \circ g$	Open of f by g
$f \bullet g$	Close of f by g
$f \hat{\circ} g$	Open top-hat of f by g
$f \hat{\bullet} g$	Close top-hat of f by g
$\breve{f}$	Reflection of f
$ASF^n_{oc,g}(f)$	Alternating sequential filter of stage n, closing and opening
$f \oplus_g D$	Conditional dilation of f conditioned to g with connectivity given by the structuring element D
$f \ominus_g D$	Conditional erosion of f conditioned to g with connectivity given by the structuring element D
$(f \oplus_g D)^n$	N-conditional dilation of f conditioned to g with connectivity given by the structuring element D
$(f \ominus_g D)^n$	N-conditional erosion of f conditioned to g with connectivity given by the structuring element D
$f \ \Delta_D \ g$	Inf-reconstruction of f from the marker g using the connectivity given by the structuring element D
$f \nabla_D g$	Sup-reconstruction of f from the marker g using the connectivity given by the structuring element D
$f \circ_D g$	Reconstructive opening by the structuring element g using the connectivity given by the structuring element D
$f \bullet_D g$	Reconstructive closing by the structuring element g using the connectivity given by the structuring element D

$f \hat{\circ}_D g$	Reconstructive opening top-hat by the structuring element g using the connectivity given by the structuring element D
$f \hat{\bullet}_D g$	Reconstructive closing top-hat by the structuring element g using the connectivity given by the structuring element D
$\Lambda(A)_D$	Labeling of a binary image A with the connectivity given by the structuring element D
$f \circ (D)_a$	Area open of D-connected components of area less or equal than a
$f \bullet (D)_a$	Area close of D-connected holes of area less or equal than a
$ASF^n_{oc,g,D}(f)$	Reconstructive alternating sequential filter of stage n, closing and opening, connectivity given by D
$X_t(f)$	Threshold of image f at level t
$grad_{g1,g2}(f)$	Morphological gradient of f with the external structuring element $g1$ and internal structuring element $g2$
$HMAX_{h,D}(f)$	H-maxima of image f with contrast h and the connectivity given by D
$HMIN_{h,D}(f)$	H-minima of image f with contrast h and the connectivity given by D
$RMAX_D(f)$	Regional maximum of image f with the connectivity given by D
$RMIN_D(f)$	Regional minimum of image f with the connectivity given by D
$A \circledast T$	Hit-or-miss of binary image A by the template T
$A \odot T$	Thinning of binary image A by the template T
$A \odot_n T$	N-thinning of binary image A by the template T