# Astronomical Telescopes, Instruments, and Systems 

# Fiber-optic gyro location of dome azimuth 

John W. Kuehne

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John W. Kuehne*<br>McDonald Observatory of The University of Texas at Austin, 82 Mount Locke Road, Fort Davis, Texas 79734, United States


#### Abstract

The 2.1-m Otto Struve Telescope, world's second largest in 1939, today has modern motion control and superb tracking, yet the 19-m-diameter Art Deco dome has resisted many attempts to record its azimuth electronically. Demonstrated in January 2016, a small tactical-grade fiber-optic gyro located anywhere on the rotating structure, aided by a few fiducial points to zero gyro drift, adequately locates the azimuth. The cost of a gyro is practically independent of dome size, offering an economical solution for large domes that cannot be easily encoded with conventional systems. The $100-\mathrm{Hz}$ sampling is capable of revealing anomalies in the rotation rate, valuable for preventive maintenance on any dome. I describe software methods and time series analysis to integrate angular velocity to dome azimuth; transformation of telescope hour angle and declination into required dome azimuth, using a formula that accounts for a cross-axis mount inside an offset dome; and test results. © The Authors. Published by SPIE under a Creative Commons Attribution 3.0 Unported License. Distribution or reproduction of this work in whole or in part requires full attribution of the original publication, including its DOI. [DOI: 10.1117/1.JATIS.2.3.037001]


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## 1 Introduction

For automation, observing efficiency, and safety of both telescope and personnel, the electronic acquisition of dome azimuth is necessary. Presumably, new telescope domes are designed to be encoded reliably and inexpensively, but an older telescope like the $2.1-\mathrm{m}$ Otto Struve Telescope (OST) may not be so easily retrofitted. The drive cable can slip in its sheave or slide through the grip blocks on the dome perimeter, preventing simple encoding of the motor or cable. The only available location for encoding the perimeter is the tight space shared with the dome rail, wheels, drive cable, and high-voltage slip rings. Maintenance and repair procedures, radial and vertical variations in the dome as it rotates, grease and oil drips, monsoon rains that penetrate the base of the dome, and even insects have foiled previous attempts to use this location for barcodes, radio tags, and possibly contact rollers or a switch system. A global positioning system requires multiple antennas outside the steel dome with the added risk of lightning strikes.

With the advent of reliable, accurate, and affordable fiberoptic gyros (FOGs), the azimuth can be determined from a single point anywhere inside the dome, safe from environmental and mechanical challenges. Whereas the cost of traditional encoder systems is proportional to the dome circumference, a tactical-grade FOG has enough accuracy to be considered a constant cost, making it economical for large domes. A valuable benefit of the FOG solution for reliability-centered maintenance ( RCM ) and repair procedures is the high-sample rate ( 100 Hz in this application), which reveals anomalies in rotation due to the interaction of cable, guideway, and rail. Even for modern domes successfully encoded with low-resolution sensors, the potential RCM benefits of the gyro solution may alone be worthwhile.

The main shortcoming with an FOG is drift due to bias instability (the random variation in zero-velocity offset inherent in the gyro) and random walk of the integrated velocity, described

[^0]below. However, the normal operation of many domes, including OST, involves motion for short periods, typically 15 s every 10 min or so depending on the hour angle and declination. The key idea in this project is to integrate the velocity time series only when the dome is moving and to continually compute the bias offset when it is stationary, thereby greatly reducing drift. The remaining shortcomings are the need to know the starting position at power-up and to zero the drift through the use of one or more fiducial points depending on gyro accuracy. In the case of OST, previous failures to produce a robust solution using traditional encoders make these shortcomings appear tractable.

## 2 Gyro Operation for a Dome

An FOG uses the Sagnac effect to detect rotation rate relative to an inertial reference frame. ${ }^{1}$ Field units are compact and rugged and are widely used for routine navigation, guidance, and platform stabilization. High-performance navigational gyros have milliarcsecond resolution. Theory of operation and applications are described in Ref. 2.

The best place for an FOG is anywhere safe and convenient on the dome, with its rotation axis parallel to the dome's axis. The entire gyro system, including a small control computer, wireless transceiver, and power supplies, fits inside something the size of a lunchbox. The FOG selected below draws typically 1.25 W , and a $15-\mathrm{W}$ system can be readily assembled from consumer electronics. For high reliability, the system should be connected to an uninterruptible power supply.

### 2.1 Accuracy Requirements and Gyro Selection

In practice, observers position the OST dome manually using the 3 -deg marks on the perimeter with the value given by the telescope control system or just sighting by eye, adequate for the 30 -deg dome slit. Unfortunately, they must do this every 10 min or so, entering the dome or assisted by a video camera mounted on the telescope. The dome may repeatedly traverse
the meridian north or south as targets rise and set or may pass all points of the compass on the way to new targets, accumulating perhaps 30 min of motion from dusk to dawn. The dome rotates about $0.8 \mathrm{deg} / \mathrm{s}$, and daytime maintenance operations may require a complete turn taking 8 min .

A gyro system must be able to match the accuracy observers now attain with the 3-deg marks, and I adopted the goal of 1-deg accuracy under normal observing conditions. However, the position calculated from gyro velocity is subjected to drift depending on gyro accuracy, and therefore at least one fiducial point is required to reset the azimuth before exceeding the 3-deg limit. In this demonstration, the azimuth is reset at four locations on the points of the compass. Maintenance of four such points is relatively easy and inexpensive, and the temporary loss of a point does not disable the system, although it may degrade its accuracy. I used reed switches connected to the OST motion controller, activated by a powerful neodymium magnet on the dome.

Specifications for FOGs are standardized in IEEE Std 952. I examined bias offset, bias instability, and angle random walk derived from Allan variance measurements to help ensure that a tactical-grade gyro would meet the 1-deg accuracy goal with four fiducial points, described below.

Bias offset: Upon starting the gyro, the average velocity at rest may not be zero. Some of this bias offset is internal to the gyro's operation, and the remainder comes from earth rotation scaled by the cosine of the angle between earth's rotation axis and the gyro axis. In this application, bias offset is not critical, as there is ample time to get a good estimate before the dome moves, and the angle between the FOG and earth's axis is essentially constant.

Bias instability: Variation in bias at constant temperature measured in deg /h is usually quoted as the lowest value in the Allan variance curve. Without resorting to rigorous analysis or simulations, in the worst case, if no fiducial points were passed all night, either because the points failed or the dome remained in one quadrant, and the dome was somehow moved for 1 h , it seems reasonable to specify at least $1 \mathrm{deg} / \mathrm{h}$ to attain the accuracy goal of 1 deg .

Angle random walk: At rest, the gyro velocity time series exhibits random fluctuations measured in deg $/ \sqrt{\mathrm{h}}$, whose integration produces an unbounded random walk. Tactical FOGs can exhibit $<0.1 \mathrm{deg} / \sqrt{\mathrm{h}}$, and even an hour of motion during the night would not contribute significant azimuth error at this level.

I chose the KVH Industries DSP-3000, a small rugged unit with $100-\mathrm{Hz}$ digital output of angular velocity estimates on a standard serial interface. The bias offset of $\pm 20 \mathrm{deg} / \mathrm{h}$ is not a drawback as the offset is easily measured before the dome moves. The bias instability is $\leq 1 \mathrm{deg} / \mathrm{h}$, and the angle random walk is $\leq 0.067 \mathrm{deg} / \sqrt{\mathrm{h}}$. The maximum input rate of $\pm 375 \mathrm{deg} / \mathrm{s}$ is adequate to capture irregular dome motion, impulses, and vibrations for an RCM program.

Less costly and much smaller gyros based on microelectromechanical systems (MEMS) vibration sensors are challenging FOGs in some applications, but generally these MEMS gyros have larger bias instability and random walk, and in the foreseeable future, FOGs will outperform MEMS. ${ }^{3}$ Low-power consumption and miniaturization are the key attributes for MEMS that are unimportant in this application.

A comparison with a continuous barcode system, also capable of measuring position and velocity with high accuracy
and bandwidth, shows how affordable the FOG solution may be. A quote for an industrial barcode reader with 60 m of tape is about 2100 USD, and the DSP-3000 is about 4200 USD. Both systems require power supplies, control computer, software programming, and integration with the telescope control system, but the barcode system also requires 60 m of mounting hardware on the dome perimeter. For OST, this would mean employing expensive engineering support to design, fabricate, and install a trackway alongside the high-voltage slip rings or cable guideway. In comparison, the gyro system, completely contained in a single enclosure except for a few fiducial points, has negligible installation costs.

### 2.2 Data Acquisition and Integration

The key idea in this project is to integrate the velocity time series only when the dome is moving and to continually compute the bias offset when it is stationary. This is accomplished by setting up nonoverlapping $1-\mathrm{s}$ motion detection and $60-\mathrm{s}$ bias offset buffers, taking care to prevent contamination of the bias buffer with motion data. The size of the motion buffer and its threshold are set experimentally to avoid false positives, for example tremors of the dome caused by high winds, although occasionally integrating a few seconds at zero velocity would not contribute significant error. While motion is not detected, the bias offset is continually estimated as the average in the buffer. When motion is detected, the velocity data including the contents of the motion buffer are diverted to the integrator described below using the last computed bias offset, which remains constant during the integration. After motion ceases in the detection buffer and the integration is finished, the bias buffer starts to fill with fresh data.

Although integration could also be initiated when the dome is commanded to move by the telescope control system, seasonal winds $>150 \mathrm{~km} / \mathrm{h}$ have been known to rotate the dome of the $2.7-\mathrm{m}$ Harlan J. Smith telescope despite brakes, and OST has original switches on the bridge crane, sometimes used during maintenance procedures, that bypass computer control. However unpleasant to contemplate, a dome could rotate unbidden. For these reasons, the gyro system should not rely on dome commands for its operation.

Rectangular integration of time series by running sum is erroneous for signal variations anywhere near the Nyquist frequency. For gyro data rates upward of 100 Hz , rectangular integration for sluggish things like domes is certainly acceptable. However, for gyros with a much lower data rate, or for any application in which the full accuracy of the gyro is important, a better integration method is required, and at any rate, it is desirable to use a mathematically correct method instead of ad hoc procedures.

In a paper hardly known outside of geophysics, Peacock ${ }^{4}$ derives an optimum digital filter for discrete integration whose accuracy can be adjusted to the bandwidth of the input signal, and which has no phase distortion. Based on Bracewell's ${ }^{5}$ observation that integration is equivalent to convolution with the Heaviside step function, the filter is expressed as the usual recursive integration plus an infinite nonrecursive (convolutional) part, truncated and tapered to meet the desired accuracy requirements.

Summarizing the filter in convenient terms, let $v_{s}$ be the velocity time series indexed by $s, \Delta t$ be the sample interval, $2 K+1$ be the length of the truncated convolutional filter, and $Q$ be the term that suppresses Gibbs ringing at the expense of
roll-off frequency. Then the estimated position $p_{s}$, within the constant of integration, is
$p_{s}=r_{s}+c_{s} \quad r_{s}=r_{s-1}+v_{s} \Delta t \quad c_{s}=\sum_{k=-K}^{K} g_{k} w_{k} v_{s-k} \Delta t$,
with
$g_{k}=\frac{1}{2}-1[k \geq 0]+\frac{1}{\pi}\left[\pi k-\frac{(\pi k)^{3}}{3 \cdot 3!}+\frac{(\pi k)^{5}}{5 \cdot 5!}+\cdots\right]$
$w_{k}=\left[1-(k / K)^{2}\right]^{Q}$.
Peacock gives both theoretical and practical guidance for selection of $K$ and $Q$, with an example of $K=10$ and $Q=3$ performing with just $1 \%$ error at $86 \%$ of the Nyquist frequency. Additionally, care must be exercised computing $g_{k}$ as the series converges slowly, and for $K>10,64$-bit arithmetic may be inadequate. Note that $c_{s}$ takes $K$ samples in the future (practically $K-1$, because $g_{K}=g_{-K}=0$ for $Q>0$ ), and in real-time applications, this incurs a time delay: having to wait for high accuracy must be balanced against having something less reliable immediately. This also demonstrates that after $2 K+1$ samples have passed at zero velocity, the rectangular integration matches the optimum filter.

## 3 Test Results

### 3.1 Six-Hour (90 deg) Tests

During observations, the OST dome is moved periodically to keep the tube approximately centered in the $30-\mathrm{deg}$ slit. In the experiment in Fig. 1, the dome was moved in 7.5-deg steps for 90 deg , covering about 6 h on sky. At 400 s , the dome stuck and had to be rocked back a few times to get a running
start. We believe this is caused by a sag in the rail that traps the heaviest part of the $104,000 \mathrm{~kg}$ structure holding the bridge. The drive cable rides in lead grip blocks that allow it to slip to prevent overloading, but until these mechanical problems are fixed there can be no dome automation, although it is already under software control for safety. At 1600 s , I nudged the dome until the reported azimuth was 0 deg, and in this test, there was no discernible error. Similarly at 1900 s , nudging to 90 deg brought the dome right back to its starting position.

Repeated many times over several weeks, the error moving 90 deg in steps was less than $4 \mathrm{~cm}(0.25 \mathrm{deg})$ and often smaller when rotated back to the starting position, perhaps indicating a simple scaling error. Moving the dome 360 deg (each trip is 8 min ) produced errors less than 6 cm .

On the basis of many experiments like this, gyro drift can be well controlled with four fiducial points located on the points of the compass. Typically during observations, the telescope crosses the meridian ( S or N fiducial points) where the seeing is best, whether the program is a survey involving many targets or one lasting many hours on a single target. Even if an observer somehow moved only between a pair of fiducial points, in the worst case, it would take about 360 deg of motion in that $90-\mathrm{deg}$ interval before losing 1-deg accuracy. In this unlikely scenario, the software would issue a warning of degraded accuracy, and more fiducial points or a more accurate gyro would be advised. For example, next in the series of KVH gyros is the DSP-1760, about 10 times more accurate at less than twice the price.

### 3.2 Tachometer Example for Reliability-Centered Maintenance

Many gyros are capable of data rates up to 1000 Hz , functioning as a tachometer useful for measuring variations in rotation for preventive maintenance and monitoring. The OST dome is in


Fig. 1 Counterclockwise test starting from 90 deg (slit east). (a) Azimuth integrated from velocity (b) using computed bias offsets (c).


Fig. 2 Counterclockwise test starting from 90 deg (slit east) running 360 deg, showing velocity variations related to mechanical problems that may indicate incipient failure.
dire need of permanent repair, having suffered collisions in January 2015 that broke several steel castings with explosive force, large enough to blow off radio tags that I had just installed for measuring dome azimuth! In Fig. 2, the dome is rotated 360 deg, revealing a number of mechanical problems. Most obvious is the large spike where the dome unexpectedly lurched to a slower rate. There are 24 smaller spikes where the drive cable picks up the grip blocks subject to wear on each 15 deg (1 h) dome sector. The reduced rate and sway in the first minute occur near the sector where mechanical problems encountered in Fig. 1 caused the dome to stick. Clearly, velocity variations recorded by FOG can be used to diagnose problems and guide engineering solutions.

## 4 Conclusions

I have demonstrated that a tactical-grade FOG, located anywhere on the dome out of harm's way, can adequately locate the dome azimuth and record velocity variations. The single point of installation is a huge advantage for domes where traditional encoders have a history of failure for environmental or mechanical reasons. Gyro drift, minimized by integrating only when the dome is moving using the latest bias estimate, necessitates the use of a few fiducial points, the number determined experimentally depending on the gyro accuracy. The inconvenience of fiducial points is insignificant compared with the installation of systems like barcodes, which must festoon the entire circumference at considerable cost for large domes. With a tactical-grade gyro similar to the unit tested, four fiducial points will suffice, with perhaps a fifth bracketing the normal stow position to guarantee a good starting value. Selecting a more accurate (and expensive) gyro is an option for reducing the number of fiducial points.

I have been asked to say a few words about future plans. I hope the value of my research extends beyond the particular problems of OST, and that if you have a large dome that needs encoding for positioning or an RCM program, you would consider collaborating on an FOG solution.

## Appendix: Derivation of Dome Azimuth

For an equatorial cross-mount like OST whose tube is offset 2 m from the polar axis, the dome azimuth required to keep the tube centered in the slit is quite different from the telescope's azimuth. Additionally, the dome is raised slightly above the telescope center, and for some telescopes, the center may be shifted horizontally, e.g., the retrofitted $0.8-\mathrm{m}$ Rice University telescope at McDonald Observatory. I can find no explicit closed-form solution in the literature for an offset equatorial telescope where the dome is raised or off-center, as is the case with OST. My solution is solved and factored to remove the denominator in $L$ below and can be expressed using just four computer program statements.

Referring to Fig. 3, assume the telescope has an equatorial mount with the declination axis intersecting the polar axis at the origin. Let HA be the telescope hour angle, DEC be the declination, and LAT be the observatory latitude. Model the telescope tube at $\mathrm{HA}=0$ and $\mathrm{DEC}=90$ as the line segment from $(0, M, 0)$ to $(0, M, L)$ with $M$ the mirror/tube offset from the polar axis with positive east and $L$ the tube length. With $H=$ $\mathrm{HA} \pi / 12, D=(90-\mathrm{DEC}) \pi / 180$, and $a=-(90-\mathrm{LAT}) \pi / 180$, the following matrices rotate the tube endpoint $(0, M, L)$ first in declination, then hour angle, and finally by the observatory latitude to


Fig. 3 Starting coordinate system with tube of length $L$ offset a distance $M$ on the east side of the axis at $\mathrm{HA}=0$ and $\mathrm{DEC}=90$, modeled as the point from $(0, M, 0)$ to $(0, M, L)$. Successive rotation in declination, hour angle, and observatory latitude transforms the point to the local coordinate system. If the telescope is configured tube-west, then $M$ is negative, which may be chosen to get the tube over the lower windscreen for rising targets or to avoid collision near the meridian on tripod-mounted telescopes.

$$
\begin{align*}
& {\left[\begin{array}{ccc}
\cos (a) & 0 & \sin (a) \\
0 & 1 & 0 \\
-\sin (a) & 0 & \cos (a)
\end{array}\right]\left[\begin{array}{ccc}
\cos (H) & \sin (H) & 0 \\
-\sin (H) & \cos (H) & 0 \\
0 & 0 & 1
\end{array}\right]} \\
& \times\left[\begin{array}{ccc}
\cos (D) & 0 & \sin (D) \\
0 & 1 & 0 \\
-\sin (D) & 0 & \cos (D)
\end{array}\right]\left[\begin{array}{c}
0 \\
M \\
L
\end{array}\right]=\left[\begin{array}{l}
p_{x} \\
p_{y} \\
p_{z}
\end{array}\right] . \tag{3}
\end{align*}
$$

Now in the local coordinate system at the observatory with positive values for $x$ south, $y$ east, and $z$ up, the endpoint of the tube is
$p_{x}=\cos (a)[L \cos (H) \sin (D)+M \sin (H)]+L \sin (a) \cos (D)$
$p_{y}=-L \sin (H) \sin (D)+M \cos (H)$
$p_{z}=-\sin (a)[L \cos (H) \sin (D)+M \sin (H)]+L \cos (a) \cos (D)$.

Assume the dome is spherical with radius $R$ and $\left(c_{x}, c_{y}, c_{z}\right)$ is its center point in the local coordinate system, e.g., an eastward shift of the dome is positive $c_{y}$ and a northward shift is negative $c_{x}$. To constrain ( $p_{x}, p_{y}, p_{z}$ ) to be on the dome, solve for $L$ on the sphere $\left(p_{x}-c_{x}\right)^{2}+\left(p_{y}-c_{y}\right)^{2}+\left(p_{z}-c_{z}\right)^{2}-R^{2}=0$ and substitute back into Eq. (4). Abbreviating $c_{x}, c_{y}, c_{z}$ as the variables $x, y, z$, the Maxima ${ }^{6}$ solutions for the quadratic using solve, simplified to factor out the denominator using trigsimp, are (in Fortran, with 72-character lines wrapped for journal publication)

$$
\begin{aligned}
& \mathrm{L}=\mathrm{W}^{*} \operatorname{sqrt}\left(\mathrm{R} * * 2-\mathrm{M}^{* *} 2+\left(\left(2^{*} \cos (\mathrm{a}) * \mathrm{x}-\right.\right.\right. \\
& \left.\left.2{ }^{*} \sin (a){ }^{*} z\right)^{*} \sin (H)+2{ }^{*} Y^{*} \cos (H)\right) * M+ \\
& 1((\cos (a) * * 2-1) * z * * 2+2 * \cos (a) * \sin (a) \\
& \left.{ }^{*} x^{*} z+y^{* *} 2-\cos (a) * * 2 * x * * 2\right) * \text { si }
\end{aligned}
$$

```
\(2 \mathrm{n}(\mathrm{D}) * * 2{ }^{*} \sin (\mathrm{H}) * * 2+\left(2^{*} \sin (\mathrm{a})^{*} \mathrm{y}^{*} \mathrm{z}-\right.\)
    \(\left.2{ }^{*} \cos (\mathrm{a}){ }^{*} \mathrm{x} * \mathrm{y}\right){ }^{*} \sin (\mathrm{D}) * * 2 * \cos (\mathrm{H})\)
\(3+\left(-2{ }^{*} \cos (a){ }^{*} y^{*} z-2^{*} \sin (a){ }^{*} x^{*} y\right){ }^{*} \cos (D)\)
    \(\left.{ }^{*} \sin (\mathrm{D})\right) * \sin (\mathrm{H})+(-2 * \cos (\mathrm{a})\) *
\(4 \sin (a) * z * * 2+(4 * \cos (a) * * 2-2) * x^{*} z\)
    \(\left.+2{ }^{*} \cos (a){ }^{*} \sin (a) * x^{*} * 2\right){ }^{*} \cos (D){ }^{*} \operatorname{si}\)
\(5 \mathrm{n}(\mathrm{D}) * \cos (\mathrm{H})+((1-2 * \cos (\mathrm{a}) * * 2) * \mathrm{z} * * 2-\)
    \(4^{*} \cos (a){ }^{*} \sin (a){ }^{*} x^{*} z+(2 * \cos (a)\)
\(6 * * 2-1) * x * * 2) * \sin (D) * * 2+(\cos (a) * * 2-1)\)
    \({ }^{*} z^{*} * 2+2{ }^{*} \cos (a){ }^{*} \sin (a){ }^{*} x^{*} z-y\)
\(7 * * 2-\cos (a) * * 2 * x * * 2)-y^{*} \sin (D) * \sin (H)\)
    \(+\left(\cos (a){ }^{*} x-\sin (a){ }^{*} z\right){ }^{*} \sin (D) *\)
\(8 \cos (H)+(\cos (a) * z+\sin (a) * x){ }^{*} \cos (D)\)
```

where $W= \pm 1$ selects the root. Programmers will appreciate that the equation can be substituted into programs written in many computer languages with little editing.

In the usual case $\left(c_{x}, c_{y}, c_{z}\right)$ is $(0,0,0)$, then $L= \pm \sqrt{R^{2}-M^{2}}$. Normally the positive root substituted into Eq. (4) produces the desired point above the horizon $\left(p_{z}>0\right)$. To get the required azimuth of the dome, we need the vector from the dome center to this point, which is $\left\langle p_{x}-c_{x}\right.$, $\left.p_{y}-c_{y}, p_{z}-c_{z}\right\rangle$. Then the compass azimuth of the dome with $0=\mathrm{N}, 90=\mathrm{E}, 180=\mathrm{S}$, and $270=\mathrm{W}$ is

$$
\begin{equation*}
180\left[1-\tan ^{-1}\left(p_{y}-c_{y}, p_{x}-c_{x}\right) / \pi\right] . \tag{5}
\end{equation*}
$$

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John W. Kuehne received his BA degree in music performance and his BS degree in geology and mathematics from the University of Kentucky, graduating Phi Beta Kappa. He also received his PhD in geology (atmospheric excitation of polar motion) supervised by Clark R. Wilson from the University of Texas at Austin. He is a research engineering/scientist associate at McDonald Observatory.


[^0]:    *Address all correspondence to: John W. Kuehne, E-mail: jkuehne@utexas .edu

