

SPECKLED SPECKLE STATISTICS WITH A SMALL NUMBER OF SCATTERERS: IMPLICATION FOR BLOOD FLOW MEASUREMENT

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ABSTRACT

Formulas relating to diffraction of the focused Gaussian beam from a narrow blood microvessel have been derived. A native vessel has been considered as a set of moving random screens. The correlation function of intensity fluctuations of statistically inhomogeneous speckled speckles has been studied with applications to flow measurement. Dependencies of statistical characteristics of biospeckles with a small number of scatterers on the number of scattering screens have been analyzed. It has been shown that the value of Doppler shift in the scattered light essentially depends on the spatial velocity distribution in the blood flow. © 1998 Society of Photo-Optical Instrumentation Engineers. [S1083-3668(98)00403-1]

Keywords scattering; biospeckles; blood flow.

1 INTRODUCTION

For some human diseases parameters of blood microcirculation are changed. Alteration of blood flow characteristics and the structure of the flow must be considered to contain important pathogenetic and diagnostic information.

Very often blindness is caused by disorders of microcirculation in the vessel closed to the optic nerve. Thus, analysis of retinal blood flow may be useful in ophthalmologic diagnostics. Such analysis also allows one to evaluate the pathologies of brain microvessels. Intravascular aggregation of red blood cells also arises at several cardioidiseases. This pathology may be found through the study of blood flow in a single vessel of bulbar conjunctiva or nail bed. An essential change of the character of blood motion in the capillaries may arise at the applications of drugs. Investigation of such disorders, for example, in the vessels of rat mesentery, may serve as the basis for the screening of medical preparations or the study of the influence of toxins.¹

At the present time two main techniques of laser diagnostics of narrow native microvessels have been developed: Doppler and speckle-interferometric ones. These techniques are practically identical.² Many works have been dedicated to the study of scattering of laser irradiation in a single vessel with application to biomedical diagnostics (see, for example, Refs. 3 and 4). However, two existing problems must be outlined.

At first, the study of a narrow microvessel by means of laser requires the strong focusing of the

beam. This is the case of diffraction with a small number of scatterers and a very small number of papers have been specially dedicated to the study of this subject. Many phenomena, which are shown in this type of scattering, have not been analyzed in detail.

The second, more important problem, concerns the study of speckles, which are formed inside the considered vessel. Scattering of coherent light in the multilayered medium leads to the appearance of *speckled speckles*. To the author's knowledge, this term has not been mentioned in the papers dedicated to the scattering of light in biotissues. But, as is well known, it is not that difficult to obtain a Doppler signal in a biomedical experiment. What is more difficult is to interpret it correctly. At present, there is no effort to explain theoretically the influence of speckled biospeckles on the result of blood flow measurements in narrow capillaries.

This paper partially fills the gap in this field. The statement of the problem concerning the speckled speckles is reviewed briefly in Sec. 2. In Sec. 3, formulas relating to the scattering of the focused laser beam from single random screen are derived. Section 4 is dedicated to the investigation of statistical properties of doubly scattered speckles. Section 5 covers cascaded speckles. Obtained results have been adapted to the problem of blood flow measurement in Sec. 6.

2 WHAT ARE THE "SPECKLED SPECKLES?"

The partially coherent light scattered from a diffuse object produces so-called speckled speckles at a far zone of diffraction.⁵ Sometimes speckles from cascaded diffusers may be called "cascaded speckles"⁶ or "doubly scattered light."⁷

These speckles undergo non-Gaussian statistics. Intensity fluctuations of speckled speckles in the Fraunhofer zone are approximately K distributed.^{7,8}

First-order statistics of doubly scattered image speckles have been analyzed in Refs. 6 and 9. These image speckles also obey K statistics, but sometimes they possess specific properties. In particular, it has been shown⁹ that the fluctuations of the doubly scattered amplitude are not K distributed when the second diffuser is motionless. In this case image speckled speckles amplitude obeys circular Gaussian statistics.

Some correlation properties of doubly scattered light have been discussed in Refs. 7 and 10–16. It has been indicated⁷ that the doubly scattered light is not an ergodic process. Formulas of the intensity correlation function of speckled speckles has been derived in Ref. 10. The case of scattering from a number of moving phase screens has been studied in the paper¹⁵ by Okamoto and Asakura.

It is interesting to note that the concept of "speckled speckles" was introduced in Ref. 5, but the subject has a longer prehistory (see Refs. 17 and 18 and their references). Analysis of double propagation of light through a random medium is important from the viewpoint of imaging. In coherent light scattering in double passage configuration, speckled speckles have also been obtained. Specific phenomena of backscattering enhancement or partial phase conjugation by the double passage of light through a deep phase screen may be observed.^{17–24} The optical memory effect has been studied, for instance, in Refs. 25–27. Having the same nature as enhanced backscattering, it is also caused by double passage of coherent light through a finite-thickness random slab. It should be shown that statistical properties of light which have been scattered twice by two independent screens and statistical properties of speckles in double passage configuration are rather different. However, intensity distributions in both cases are members of the K -distribution class.²⁸

It is important to emphasize that diffraction of the coherent wave from a random layered object may be interpreted as scattering from a set of phase screens. In this case, coherent scattering by multi-layered slab speckled speckles are also formed. The transport of waves through such a medium has been considered in a number of works (see, for example, Ref. 29).

Therefore, at the present time, processes of scattering from the cascade of diffusers and statistics of speckled speckles are investigated in detail but common approaches involve diffraction of waves

with a broad front. As was already mentioned, high-resolution measurements usually are carried out with focusing optics.

In strongly focused laser beam scattering from a random (even single) screen, a special type of speckle has been observed. Such speckle statistics with a small number of scatterers have already been investigated.^{31–36} Non-Gaussian scattering also occurs in the case of random diffraction of the strongly focused coherent beam.³¹ The speckle field formed with a small number of scatterers is a statistically inhomogeneous process.³⁴ The dynamics of such speckles are characterized by the new type of manifestation of the Doppler effect.^{37–39}

As far as the author knows, there are no works dedicated to the analysis of correlation properties of speckled speckles with a small number of scatterers. Only the first order statistics of such speckles have been studied in Ref. 30. In the present paper, formulas relating to diffraction of a focused Gaussian beam from a set of phase screens are derived.

3 SINGLE SCATTERING OF STRONGLY FOCUSED GAUSSIAN BEAM FROM RANDOM PHASE SCREEN

Let us analyze the correlation properties of dynamic speckles in the Fresnel zone. A one-dimensional notation is used throughout. The focused Gaussian beam is moving along a perfectly transmitted deep phase screen. At normal beam incidence the scattered complex amplitude $U^s(X)$ takes the form

$$U^s(X) = \text{const}_1 \cdot \int_{-\infty}^{\infty} U_v(X - \alpha) \cdot G(\alpha) d\alpha, \quad (1)$$

where

$$G(\alpha) = \exp \left[- \left(\frac{1}{W_1^2} - \frac{\pi \cdot i \cdot Z_1}{Z_1^2 + \pi^2 \cdot W_0^4} \right) \cdot \alpha^2 \right] \times \exp \left(\frac{\pi \cdot i \cdot (X_2^0 - \alpha)^2}{\Delta Z_1} \right), \quad (2a)$$

$$W_1 = W_0 \cdot \left[1 + \left(\frac{Z_1}{\pi \cdot W_0^2} \right)^2 \right]^{0.5}, \quad (2b)$$

where W_1 is the radius of the beam in the scattering plane; Z_1 is the distance between the scattering plane and waist beam plane; and W_0 is the waist beam radius. X_2^0 [see Figure 1(a)] is the fixed speckle observation point coordinate in the mobile coordinate system (X_{mv}, Y_{mv}, Z_{mv}) connected with the beam; X is the waist center coordinate of the moving beam, illuminating the investigated screen; ΔZ_1 is the distance between the scattering (\bar{X}_1) and observation (\bar{X}_2) planes. When the phase screen is

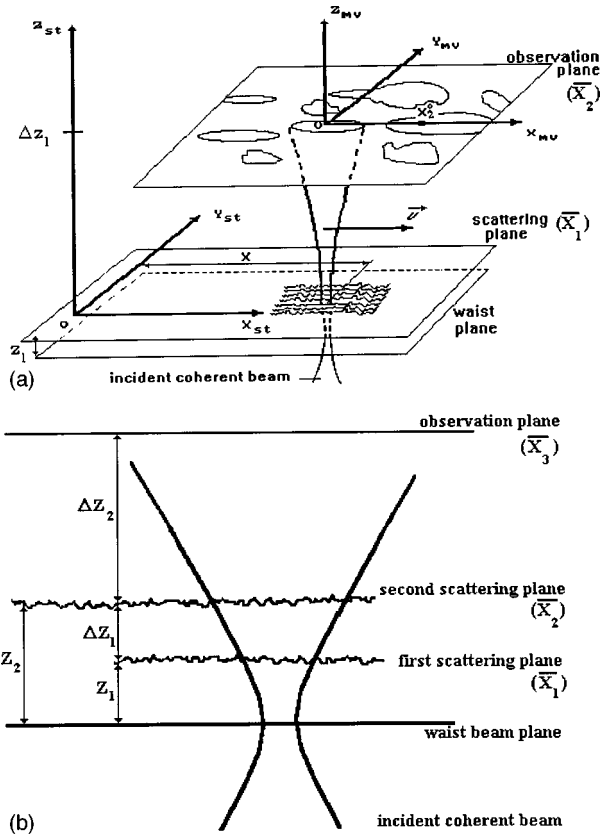


Fig. 1 Optical system under consideration. (a) Dynamic scattering from a single screen; (b) illustration of the process of formation of dynamic speckled speckles.

illuminated by a plane wave of unit amplitude, the complex amplitude $U_v(X)$ behind the phase screen is given by:

$$U_v(X) = \exp[-2\pi i \cdot H(X)], \quad (3)$$

where $H(X)$ is the screen profile function. In Eqs. (1)–(3) all the length dimensions are normalized to the light wavelength λ and the amplitudes of all the fields are normalized to the field amplitude in the waist center of the focused Gaussian beam. Thus, in Eqs. (1)–(3) all variables are dimensionless, and const_1 is an unimportant constant which may further be ignored. The screen profile $H(X)$ is assumed to be determined by stationary Gaussian stochastic process defined by the properties $\langle H(X) \rangle = 0$, $\langle H(X) \cdot H(X') \rangle = \sigma^2 \exp[-(X-X')^2/L_c^2]$, where the angle brackets denote an average over the ensemble of possible screen profiles, while $\sigma^2 = \langle H(X)^2 \rangle$ is the mean-square departure of the phase screen surface from flatness, and L_c is the correlation length of the screen profile.

As follows from the results presented in Refs. 40 and 41 for the case of plane wave diffraction from a deep phase screen, the correlation function of the amplitude U_v [in the (\bar{X}_1) plane] is described by an expression of the following form:

$$\Gamma_v^{(1)}(\xi) = \exp[-A_{1,v} \cdot \xi] \quad (4)$$

(where $A_{1,v} = [4\pi^2 \cdot \sigma^2 / L_c^2]$) for the screen under consideration).

Using Eqs. (1)–(4), we will obtain the formula for temporal correlation functions of the amplitude fluctuations in the scattered field. The correlation function of the kernel of the convolution of Eq. (1) is defined by:^{36,42}

$$B_g(\xi) = \int_{-\infty}^{\infty} G(\alpha) \cdot G^*(\alpha + \xi) d\alpha. \quad (5)$$

It is not difficult to show that the autocorrelation function of amplitude fluctuation in the observation plane has the form:⁴²

$$\Gamma_u^{(1)}(\xi) = \Gamma_v^{(1)} \otimes B_g, \quad (6)$$

and \otimes is a convolution symbol.

After ordinary transformations,⁴³ Eq. (6) becomes

$$\Gamma_u^{(1)}(\xi) = \text{const}_2 \times \exp \left\{ - \frac{A_{1,v} \cdot \left[\left(\frac{1}{W_1} \right)^4 + \Psi_1^2 \right]}{2 \cdot A_{1,v} \cdot \left(\frac{1}{W_1} \right)^2 + \left(\frac{1}{W_1} \right)^4 + \Psi_1^2} \cdot \xi^2 \right\} \cdot \exp \left\{ - \frac{2 \cdot A_{1,v} \cdot \left(\frac{1}{W_1} \right)^2 \cdot Q}{2 \cdot A_{1,v} \cdot \left(\frac{1}{W_1} \right)^2 + \left(\frac{1}{W_1} \right)^4 + \Psi_1^2} \cdot i \cdot \xi \right\}, \quad (7)$$

where

const_2

$$= [\text{const}_1]^2 \cdot \frac{\pi}{\left[2 \cdot A_{1,v} \cdot \left(\frac{1}{W_1} \right)^2 + \left(\frac{1}{W_1} \right)^4 + \Psi_1^2 \right]^{0.5}} \cdot \exp \left\{ - \frac{\left(\frac{1}{W_1} \right)^2 \cdot Q^2}{4 \cdot A_{1,v} \cdot \left(\frac{1}{W_1} \right)^2 + 2 \cdot \left(\frac{1}{W_1} \right)^4 + 2 \cdot \Psi_1^2} \right\}, \quad (8a)$$

$$\Psi_1 = - \left(\frac{\pi \cdot Z_1}{Z_1^2 + \pi^2 \cdot W_0^4} + \frac{\pi}{\Delta Z_1} \right), \quad (8b)$$

$$Q = \frac{2 \cdot \pi \cdot X_2^0}{\Delta Z_1}. \quad (8c)$$

If $\sigma \gg 1$, then the last exponent in Eq. (7) becomes

$$\exp\left(-\frac{2 \cdot A_{1,v} \cdot \left(\frac{1}{W_1}\right)^2 \cdot Q}{2 \cdot A_{1,v} \cdot \left(\frac{1}{W_1}\right)^2 + \left(\frac{1}{W_1}\right)^4 + \Psi_1^2}\right) \cdot i \cdot \xi$$

$$= \exp(-Q \cdot i \cdot \xi). \tag{9}$$

This means that the average phase in the scattered field approaches the phase profile in a nondisturbed Gaussian beam as σ essentially exceeds 1. Equation (7) also shows that the correlation of the scattered amplitude is expressed by a Gaussian function.

4 DOUBLY SCATTERED DYNAMIC SPECKLES

Let us place the second moving phase screen in the plane (\bar{X}_2) [see Figure 1(b)]. Clearly, this screen is suffused with dynamic speckles. The second screen is moving with the same velocity as the first screen. When an unlimited wave (having amplitude-phase modulation) scatters from phase screen, the correlation function of the amplitude of the speckles behind the second screen is expressed by the relation

$$\Gamma_v^{(2)}(\xi) = \exp(-A_{z,v} \cdot \xi^2) \tag{10}$$

where

$$A_{2,v} = \frac{A_{1,v} \cdot \left[\left(\frac{1}{W_1}\right)^4 + \Psi_1^2\right]}{2 \cdot A_{1,v} \cdot \left(\frac{1}{W_1}\right)^2 + \left(\frac{1}{W_1}\right)^4 + \Psi_1^2} + A_2^0; \tag{11a}$$

value of A_2^0 is defined by statistical properties of the second phase screen. The screens under consideration are identical; then

$$A_2^0 \equiv A_{1,v} = \frac{4 \pi^2 \cdot \sigma^2}{L_c^2}. \tag{11b}$$

Presently, let us take into account that the front of the incident wave is limited, i.e., [as follows from Eq. (7)] the profile of averaged intensity in the falling beam is described by the Gaussian function:

$$I(X_2) = \text{const}_2 \cdot \exp\left(-\frac{(X_2)^2}{(W_2)^2}\right) \tag{12a}$$

where

$$W_2 = \left(\frac{2 \cdot A_{1,v} \cdot \left(\frac{1}{W_1}\right)^2 + \left(\frac{1}{W_1}\right)^4 + \Psi_1^2}{\left(\frac{2 \pi^2}{\Delta Z_1^2}\right) \cdot \left(\frac{1}{W_1}\right)^2}\right)^{0.5}. \tag{12b}$$

Then, if doubly scattered light is observed in the new observation plane (\bar{X}_3), the correlation function of the amplitude of speckled speckles takes the form

$$\Gamma_u^{(2)}(\xi) = \exp(-A_{2,u} \cdot \xi^2), \tag{13}$$

where

$$A_{2,u} = \frac{A_{2,v} \cdot \left[\left(\frac{1}{W_2}\right)^4 + \Psi_2^2\right]}{2 \cdot A_{2,v} \cdot \left(\frac{1}{W_2}\right)^2 + \left(\frac{1}{W_2}\right)^4 + \Psi_2^2}, \tag{14a}$$

$$\Psi_2 = -\left(\frac{\pi \cdot Z_2}{Z_2^2 + \pi^2 \cdot W_0^4} + \frac{\pi}{\Delta Z_2}\right), \tag{14b}$$

Z_2 is the distance between the (\bar{X}_2) plane and waist beam plane; ΔZ_2 is the distance between the (\bar{X}_2) plane and new observation plane (\bar{X}_3).

5 SPECKLES SCATTERED FROM A SET OF MOVING RANDOM SCREENS

It is not difficult to obtain recurrent formulas corresponding to the case of beam scattering from a number of identical phase screens. The screens move with equal velocities. For the case of diffraction from the cascade of n equidistant screens, the correlation function of amplitude of scattered light is given by

$$\Gamma_u^{(n)}(\xi) = \exp[-A_{(n)} \cdot \xi^2], \tag{15}$$

where

$$A_{(n)} = \frac{A_{(n-1)} \cdot \left[\left(\frac{1}{W_{(n-1)}}\right)^4 + \Psi_{(n)}^2\right]}{2 \cdot A_{(n-1)} \cdot \left(\frac{1}{W_{(n-1)}}\right)^2 + \left(\frac{1}{W_{(n-1)}}\right)^4 + \Psi_{(n)}^2} + \frac{4 \pi^2 \cdot \sigma^2}{L_c^2}, \tag{16a}$$

$$W_{(n)} = \left[\frac{2 \cdot A_{(n-1)} \cdot \left(\frac{1}{W_{(n-1)}}\right)^2 + \left(\frac{1}{W_{(n-1)}}\right)^4 + \Psi_{(n)}^2}{\left(\frac{2 \pi^2}{\Delta Z_{(n)}^2}\right) \cdot \left(\frac{1}{W_{(n-1)}}\right)^2}\right]^{0.5}, \tag{16b}$$

$$\Psi_{(n)} = -\left[\frac{\pi \cdot Z_{(n)}}{(Z_{(n)})^2 + \pi^2 \cdot W_0^4} + \frac{\pi}{\Delta Z_{(n)}}\right], \tag{16c}$$

where $Z_{(n)}$ is the distance between the n screen and the waist beam plane, $\Delta Z_{(n)}$ is the distance between the (n) and ($n + 1$) screens.

Equations (15)–(16) are the main theoretical results of the article.

6 DISCUSSION OF RESULTS

First, the evident result should be stated. As follows from Eq. (16) and the procedure of normalization [see expression (18) from Ref. 36], the bandwidth of the spectrum of intensity fluctuations of biospeckles linearly depends on the velocity of moving scatterers. It has been predicted earlier by all the developed theories of light scattering in the laminar random flow (diffusing wave spectroscopy, dynamic light scattering, speckle interferometry, Monte Carlo simulation, theory of laser Doppler system) and verified experimentally.

Now let us consider the process of formation of speckled speckles inside of blood flow. Laminar flow of scattering fluid may be considered as a set of moving random screens.¹⁵ A more simple optical model of thin blood vessel (vessel has been imitated by only one moving phase screen) has been suggested and verified in Refs. 1 and 37–39, but the usage of this model does not allow one to take into account the influence of speckles formed in the scattering volume on the characteristics of the measuring signal. This may be done by means of Eqs. (15)–(16) and the results presented in Ref. 36.

It should be mentioned again that the case of a small number of scatterers is considered here. This means that parameter W_0/L_c (a characteristic number of scatterers in the single screen) is comparable with the unit. It is important to note that average speckle size S_z and decay time R of correlation function of speckles intensity fluctuations in the far zone of diffraction have been normalized on their maximal values (the maximal values correspond to the case of scattering from a single screen). Averaged intensity I on the axis of the optical system has been normalized on the intensity in the nondisturbed Gaussian beam.

6.1 CORRELATION PROPERTIES OF BIOSPECKLES INSIDE THE MICROVESSEL

Let us suppose that the distances between the screens (which are modeling the vessel) equals the value of correlation length L_c . So, the total number of screens N may be interpreted as the diameter D of the blood vessel, i.e., $D = N \cdot L_c$. The calculations have been carried out for the range of parameters $L_c > 2$, $\sigma > 0.1$ and $W_0/L_c > 1$. Evidently, the total number of scatterers in the considered system is proportional to the value of $N \cdot W_0/L_c$. As the calculation has shown, the dependencies of R , S_z , and I on the number of scattering screens are essential, see Figures 2 and 3. Average speckle size, correlation time of speckles intensity fluctuations, and averaged intensity of scattered light decrease with the growing number of scattering screens (i.e., thickness of the vessel). Clearly, such behavior of the mentioned characteristics may be observed at the increasing of scattering in all the random media.

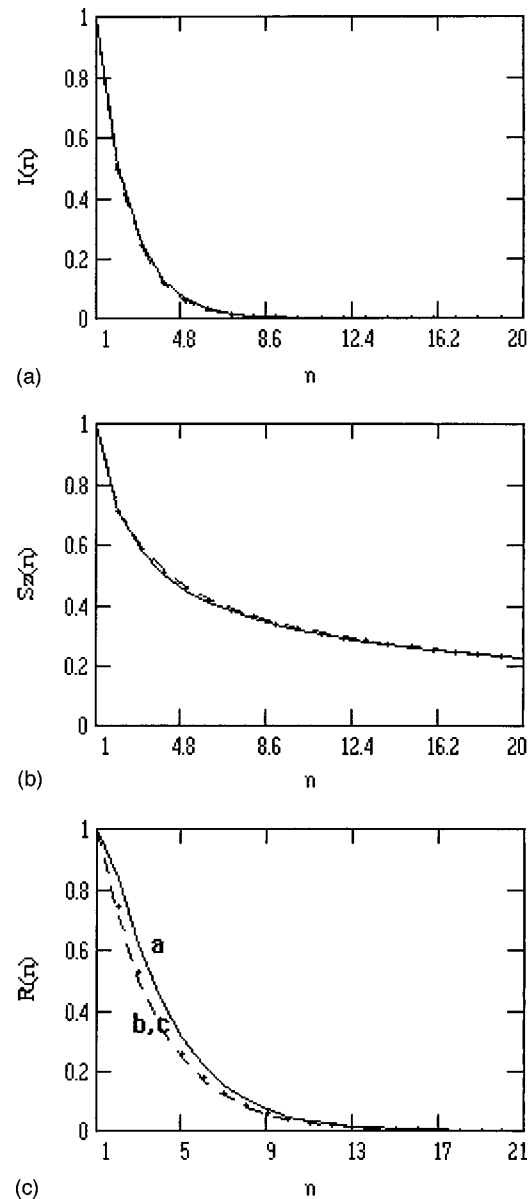


Fig. 2 Dependencies of statistical characteristics of speckled speckles on the number of scatterers and on the number of scattering screens, imitating the vessel. $L_c = 5$, $\sigma = 0.1$, beam is focused on the front wall of the vessel, total number of screens $N = 20$. (a) Normalized average intensity in the asymptotic region. (b) Normalized average speckle size. (c) Normalized decay time of correlation function: curve (a) $W_0/L_c = 1$, curve (b) $W_0/L_c = 3$, curve (c) $W_0/L_c = 5$.

At the same time, the values of R , S_z , and I practically do not depend on the parameter W_0/L_c (see Figure 2) and on other characteristics L_c and σ of single screens as well (see Figure 3). Only the weak dependence of the form of the curve $R(N)$ on the characteristics of the bio-object shows up in the case of moderate scattering [$\sigma \leq 0.1$, see Figure 3(c), curve a] and in the case of a number of scatterers less than unit [$W_0/L_c \leq 1$, see Figure 2(c), curve a].

Dependence of the form of curve I and S_z on the depth of focusing of the beam into the vessel cannot

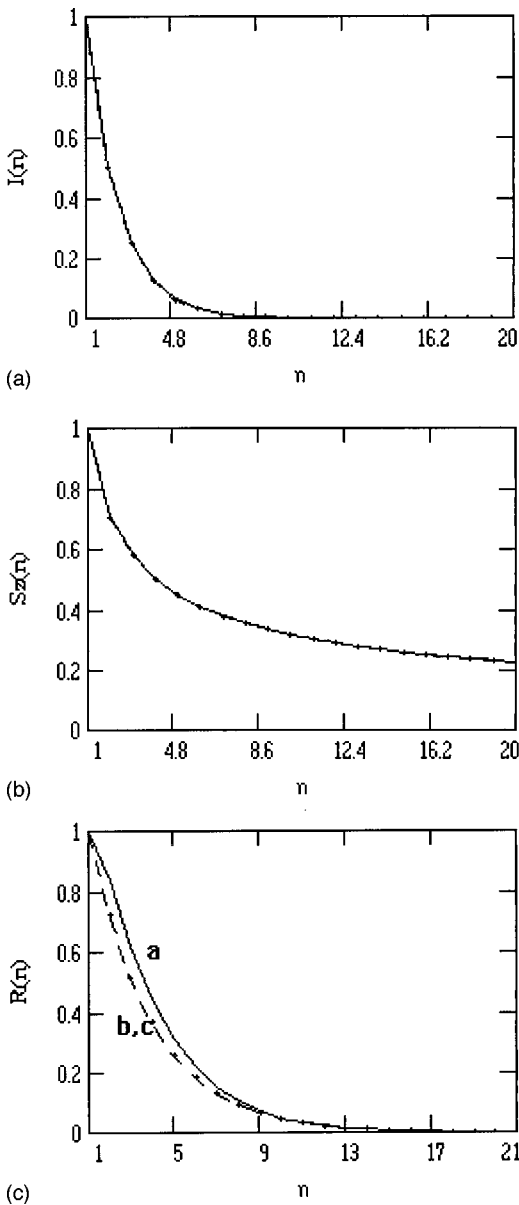


Fig. 3 Dependencies of statistical characteristics of speckled speckles on the level of scattering and on the number of scattering screens. $L_c=5$, $W_0/L_c=1$, laser beam is focused on the front wall of the vessel, total number of screens $N=20$. (a) Normalized average intensity in the asymptotic region. (b) Normalized average speckle size. (c) Normalized decay time of correlation function: curve (a) $\sigma=0.1$, curve (b) $\sigma=0.2$, curve (c) $\sigma=0.3$.

be defined. The function of **R** depends on the depth of focusing very weakly, see Figure 4(c). A very small difference in the behavior of the curves of **R** at the focusing on the front wall of the vessel may be observed.

6.2 FRAUNGHOFER SPECKLED BIOSPECKLES

If the characteristics of speckles formed in the vicinity of scattering volume are known, it is possible to study statistical properties of scattered field fluctuations in the far zone of diffraction. It has been done in Ref. 36. Following the results of the previously

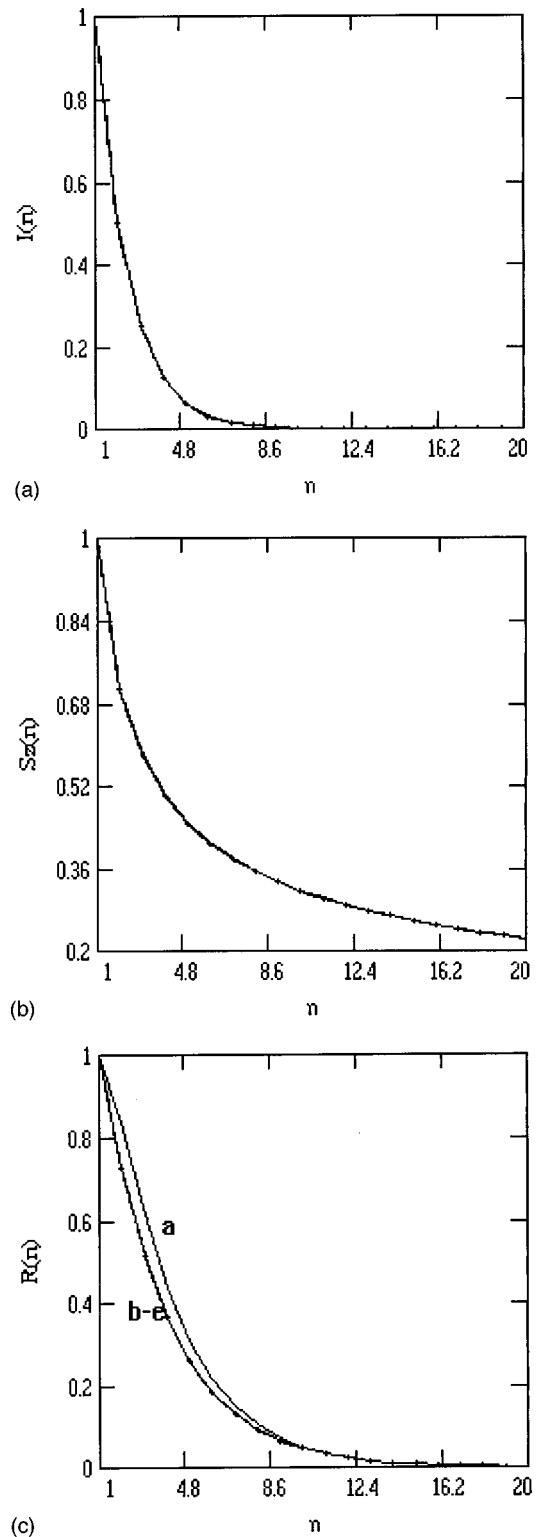


Fig. 4 Dependencies of statistical characteristics of speckled speckles on the depth of focusing. $L_c=5$, $W_0/L_c=1$, $\sigma=0.1$, $N=20$. (a) Normalized average intensity in the asymptotic region. (b) Normalized average speckle size. (c) Normalized decay time of correlation function: curve (a) the number of screens, on which the beam is focused, $M=1$; curve (b) $M=5$; curve (c) $M=10$; curve (d) $M=15$; curve (e) $M=20$.

mentioned paper and using Eq. (16) from this article we shall investigate the dependence of bandwidth of spectrum of intensity fluctuation of Fraunhofer biospeckles on spatial distribution of velocities of scatterers in the flow. Strictly speaking, Eq. (16) has been derived with the assumption, that all the scattering screens move with equal velocities. However, the differences of velocities of the screens can be easily taken into account in the considered optical model.

It usually assumes^{44,45} that velocity profile in laminar flow is described by the following expression:

$$v(r) = v_{\max} \left[1 - \left| \frac{r}{r_0} \right|^m \right], \quad (17)$$

where r is the distance from the vessel axis, r_0 is the radius of the vessel, v_{\max} is the maximum centerline velocity in the flow, and parameter m equals 2; it is the so-called Poiseuille's flow. However, sometimes (when non-Newtonian properties of blood are essential) index m may achieve the value of 3. Similar distribution of time-averaged velocity may be found in pulsating blood flow—see experimental works^{46–48} by Priezzhev and his co-workers.

The motion of red blood cells has been studied in the papers.^{49,50} As has been shown in Ref. 51, "train of particles" has been observed in the blood vessel with a diameter of about the erythrocyte size. When such a vessel is presented by a set of screens, then the profile of their velocities should be flat, i.e., all the screens move with equal velocities. A similar distribution of velocities arises when index m tends to infinity in the expression (17).

The situation when index m is less than 2 may arise as well. These values of m correspond to the case when the flow exists only in the central part of the capillary (it is absent in the regions close to the vessel wall). Such flows have been found in some lymph vessels.¹ The author believes that the same flow may appear in native blood vessels. For example, the diameter of the blood vessel does not change, but the flow of red blood cells localizes in the central part of the vessel at the application of histamine.⁵²

So, it is interesting to consider the diffraction of the focused Gaussian beam in the flows when parameter m varies in wide range. Dependence of spectrum bandwidth of biospeckles intensity fluctuations in the far zone of diffraction (in other words, the frequency bandwidth of the Doppler signal) on the value of m at different thickness of the vessel is presented in Figure 5. As can be seen, the spectrum bandwidth of the Doppler signal grows when the diameter of the vessel (or the number of equidistant moving screens) increases. The value of Doppler shift is smaller if the profile of velocities in the flow is more flat (as the higher the value of m). Dependence of the value of the Doppler shift on the depth of focusing of the beam into

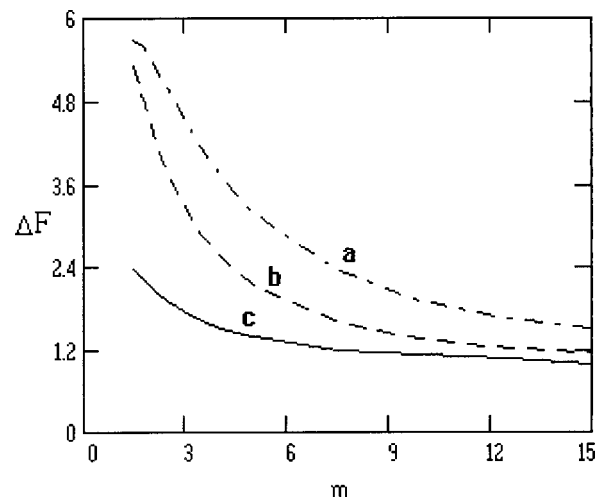


Fig. 5 Dependence of bandwidth of the spectrum of Doppler signal on the parameter of flow m and on the thickness of vessel (total number of scattering screens). $L_c=5$, $\sigma=0.1$, $W_0/L_c=3$, $M=1$; curve (a) $N=20$, curve (b) $N=15$, curve (c) $N=8$.

the vessel is presented in Figure 6. The value of ΔF depends on the conditions of very weak focusing, even in the case of a small number of scatterers. The spectrum bandwidth of fluctuations increases slightly when the beam is focused in the vessel wall, but the alteration of this value does not exceed 3%.

7 CONCLUSIONS

In this paper it has been shown that the spectrum bandwidth of the Doppler signal essentially depends on the spatial distribution of velocities in the considered bioflow. If the velocity profile is more flat than the value of the Doppler shift is smaller.

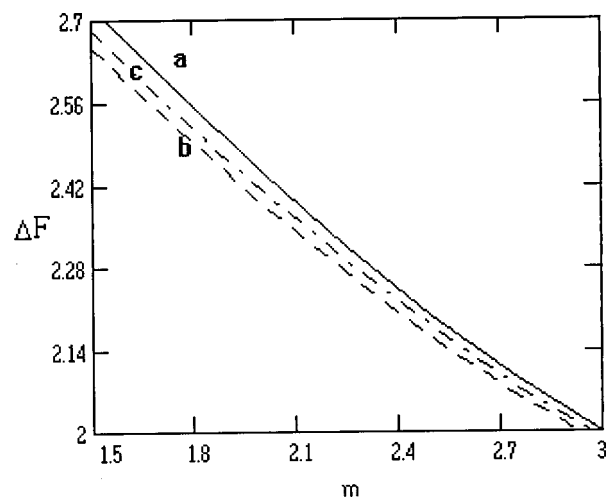


Fig. 6 Dependence of bandwidth of the spectrum of the Doppler signal on the parameter of flow m and on the depth of focusing. $L_c=5$, $\sigma=0.1$, $W_0/L_c=0.5$, $N=10$; curve (a) $M=1$, curve (b) $M=5$, curve (c) $M=10$.

The presented theory predicts an increase in the frequency bandwidth of temporal fluctuations of biospeckles intensity with an increase in diameter of the vessel. The same effect shows up with the increase in the level of scattering in the medium in the case of a small number of scatterers. These theoretical results are in good agreement with experimental data.

However, in the wide range of parameter $\sigma > 0.2$, characteristics of the measuring signal do not depend on the properties of the scattering medium. There is a simple physical explanation of this phenomenon. In this case, a strongly focused Gaussian beam may be considered as a point source. As a result, the average scattered field tends to the spherical wave. Evidently, in considering the situation, the dependence of temporal statistics of speckles on the condition of focusing and the level of scattering in the medium cannot be found.

However, if the level of scattering is not high, $\sigma \leq 0.1$, then characteristics of the measuring signal come to depend on the number of scatterers. This effect takes place only when the parameter of W_o / L_c is comparable with unit.

Blood flow in the narrow vessel is characterized by a specific profile of velocities (close to flat). The number of scatterers in such a vessel is small and, in general, unknown. These previously mentioned factors influence the result of measurements. So, laser diagnostics of blood flow in narrow capillary requires great caution.

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