Comment

Comment on the paper "Elimination of systematic error in subpixel accuracy centroid estimation"

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1 Introduction

In a paper published in 1991, Alexander and Ng[1] introduced an excellent analysis of the systematic error in centroid estimation. They used Fourier analysis to develop a closed-form equation for estimating the centroid of a shifted symmetric function. However, as we will show here, the tools they developed can be extended to address the general case of any real signal.

2 Centroid of Continuous Real Signal

Consider a continuous real signal $f(x)$. Its Fourier transform $F(s)$ is given by

$$F(s)=F_e(s)\ \exp[j\phi_o(s)]\ . \tag{1}$$

The magnitude of $F(s)$, namely $F_e(s)$, is an even function of the frequency s, and its phase $\phi_o(s)$ is an odd function. If $f(x)$ is shifted by an amount d, the Fourier transform is given by

$$F(s)=F_e(s)\ \exp\{j[\phi_o(s)-2\pi ds]\}\ . \tag{2}$$

Its derivative $F'(s)$ is given by

$$\begin{aligned}F'(s)&=F_e'(s)\ \exp\{j[\phi_o(s)-2\pi ds]\}\\&\quad+F_e(s)j[\phi_o'(s)-2\pi d]\\&\quad\times\exp\{j[\phi_o(s)-2\pi ds]\}\ .\end{aligned} \tag{3}$$

Thus, we have

$$\begin{aligned}F(0)&=F_e(0)\ \exp\{j[\phi_o(0)-2\pi d0]\}\\&=F_e(0)\end{aligned} \tag{4}$$

and similarly

$$F'(0)=F_e(0)j[\phi_o'(0)-2\pi d]\ . \tag{5}$$

Therefore, the Fourier transform representation of the centroid is given by[2]

$$\begin{aligned}c&=\frac{F'(0)}{-2\pi jF(0)}\\&=d-\frac{\phi_o'(0)}{2\pi}\ .\end{aligned} \tag{6}$$

The first term in Eq. (6) is the amount of shift applied to the signal. The second term is a phase shift that accounts for the centroid location of the unshifted real signal.

3 Centroid of Sampled Real Signal

When the signal $f(x)$ is sampled at a sampling rate T, the Fourier transform of the discrete signal is given by

$$\begin{aligned}G(s)&=\textstyle\sum_{-\infty}^{+\infty}F\left(s-\frac{n}{T}\right)\\&=\textstyle\sum_{-\infty}^{+\infty}F_e\left(s-\frac{n}{T}\right)\\&\quad\times\exp\left\{j\left[\phi_o\left(s-\frac{n}{T}\right)-2\pi\left(s-\frac{n}{T}\right)d\right]\right\}\end{aligned} \tag{7}$$

and thus

$$\begin{aligned}G'(s)&=\textstyle\sum_{-\infty}^{+\infty}F'\left(s-\frac{n}{T}\right)\\&=\textstyle\sum_{-\infty}^{+\infty}F_e'\left(s-\frac{n}{T}\right)\ \exp\left\{j\left[\phi_o\left(s-\frac{n}{T}\right)\right.\right.\\&\quad\left.\left.-2\pi\left(s-\frac{n}{T}\right)d\right]\right\}\\&\quad+\textstyle\sum_{-\infty}^{+\infty}F_e\left(s-\frac{n}{T}\right)j\left[\phi_o'\left(s-\frac{n}{T}\right)-2\pi d\right]\\&\quad\times\exp\left\{j\left[\phi_o\left(s-\frac{n}{T}\right)-2\pi\left(s-\frac{n}{T}\right)d\right]\right\}\ .\end{aligned} \tag{8}$$

Evaluating Eqs. (7) and (8) for $s=0$, and rearranging terms, we have

$$\begin{aligned}G(0)&=F_e(0)+2\textstyle\sum_1^{\infty}F_e\left(\frac{n}{T}\right)\\&\quad\times\cos\left[\phi_o\left(\frac{n}{T}\right)-2\pi\left(\frac{n}{T}\right)d\right]\end{aligned} \tag{9}$$

and

$$G'(0) = F_e(0)j[\phi_o'(0) - 2\pi d]$$

$$+ 2j\sum_1^\infty F_e'\left(\frac{n}{T}\right) \sin\left[\phi_o\left(\frac{n}{T}\right) - 2\pi\frac{n}{T}d\right]$$

$$+ 2j\sum_1^\infty F_e\left(\frac{n}{T}\right)\left[\phi_o'\left(\frac{n}{T}\right) - 2\pi d\right]$$

$$\times \cos\left[\phi_o\left(\frac{n}{T}\right) - 2\pi\frac{n}{T}d\right] . \quad (10)$$

Finally, the centroid of the sampled signal is given by

$$\bar{c} = \frac{G'(0)}{-2\pi j\, G(0)}$$

$$= d - \frac{\phi_o'(0)}{2\pi}$$

$$- \frac{\sum_1^\infty F_e'\left(\frac{n}{T}\right) \sin\left[\phi_o\left(\frac{n}{T}\right) - 2\pi\frac{n}{T}d\right]}{\pi\left\{F_e(0) + \sum_1^\infty F_e'\left(\frac{n}{T}\right) \cos\left[\phi_o\left(\frac{n}{T}\right) - 2\pi\frac{n}{T}d\right]\right\}} . \quad (11)$$

A first-order approximation of the centroid systematic error is obtained by using the first term in the numerator and neglecting the sum in the denominator relative to $F_e(0)$,[1] which results in

$$\bar{c} - c \approx \frac{F_e'\left(\frac{1}{T}\right) \sin\left[2\pi\frac{d}{T} - \phi_o\left(\frac{1}{T}\right)\right]}{\pi F_e(0)} . \quad (12)$$

Thus, as expected, the general expressions for the centroid locations, and centroid estimation error for a real signal, are similar to the equations derived for the special case of shifted symmetric signal, with additional terms accounting for the nonzero phase.

References

1. B. F. Alexander and K. C. Ng, "Elimination of systematic error in subpixel accuracy centroid estimation," *Opt. Eng.* **30**(9), 1320–1331 (1991).
2. R. N. Bracewell, *The Fourier Transform and Its Applications,* 2nd ed., McGraw-Hill, New York (1972).

Erratum

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Specification of glancing- and normal-incidence x-ray mirrors

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In this paper, an incorrect set of data was included in Table 2. The correct values are given here. These values correspond to the profile spectrum $S_1(f_x) = 6.64 \times 10^{-9}/f_x^{1.61}$ μm^3 given in Eq. (27), while those in the original table correspond to the less accurate form, $S_1(f_x) = 3 \times 10^{-8}/f_x^{4/3}$ μm^3, considered earlier in Refs. 5, 6, and 7. In either case, the values in the table are for illustration only and these changes do not affect any other results in the original paper or its conclusions. We thank L. Shao of Tucson Optical Research Corp. for bringing this error to our attention.

Table 2 Values of the Strehl decrement in percent for a surface with the straight-line spectrum in Fig. 3.

Geometry	Slope	Roughness	Total Decrement	
	Eqs. (18,21)	Eqs. (19,22)	Slope + Roughness	Exact from Eqs. (13,14)
	Low resolution (Θ = 30 μrad)			
Glancing	3.44	7.84	11.3	10.2
Normal	24.9	56.8	81.8	82.4
	High resolution (Θ = 1 μrad)			
Glancing	27.4	62.4	89.9	81.6
Normal	(199)	(453)	(651)	(656)