

Wavelet Transforms

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Introduction

Here we offer a brief review of wavelet transforms and their applications to serve as background information for the papers included in this special section. We thought it also appropriate to explain how and why these papers came to be written.

Several papers in this special section came out of conversations at the 1991 Gordon Research Conference on holography and optical computing. We all were excited about wavelet transforms and their applications. We all suspected that optics held great promise for the computationally intense tasks of wavelet transformation and inverse wavelet transformation. As evidenced by the papers that follow, everyone has their own scheme. The set of methods described below represents many of the possible approaches to optical wavelet transformation. We hope more will be invented. The field needs a variety of methods so that a person can choose the best one for their particular need.

A Brief Introduction to Wavelet Transforms

Wavelet transforms are linear and square-integrable transforms just as are the more familiar Fourier, Laplace, Hilbert, Radon, and Hadamard transforms. They are made special by their kernels called *wavelets*. Indeed, instead of a fixed kernel, we use many (possibly an infinite amount in some cases) kernels, all of which are derived from a "mother" kernel or wavelet by scale changes. Thus, in addition to "frequency" coordinates, there are shift coordinates.

The general 1-D wavelet is of the form

$$h[(x-b)/a] / \sqrt{a} ,$$

where b is the shift, a is the scale, \sqrt{a} is a normalization factor, and $h[x]$ is the mother wavelet ($b = 0$, $a = 1$). Usually $h[x]$ is of the form

$$h[x] = w(x)f(x) ,$$

where $w(x)$ is a window function (often Gaussian) and $f(x)$ is a modulation term. Both $w(x)$ and $f(x)$ are scaled and shifted.

Like Fourier transforms, wavelet transforms come in two varieties: (1) discrete wavelet transforms (DWT) in which the scale and the shift variations are discrete and (2) continuous

wavelet transforms (CWT) in which scale and shift vary continuously. With optics, we sometimes perform a hybrid wavelet transform, e.g., discrete scales and continuous shifts.

Note that for every input coordinate (space, time, etc.), there are two output coordinates. Thus, a 1-D signal produces a 2-D wavelet transform and a 2-D signal produces a 4-D wavelet transform. Thus, even "fast" wavelet transforms can be quite slow digitally. Optics may be more attractive if it can perform the wavelet transform in parallel. Furthermore, Fourier optics can map shift continuously into the lightwave complex phase information that becomes invariant under the square-law intensity detector, while a slight error in the digital computing of shift variables can produce a large error in wavelet coefficients.

In the 1-D signal case (easily generalized to N dimensions), the DWT is

$$w(a,b) = \sum_i s(x_i) h[(x_i - b) / a] / \sqrt{a} .$$

Likewise, we can invert the DWT as

$$s(x) \approx \sum_a \sum_b w(a,b) h[(x - b) / a] / \sqrt{a} .$$

While the generalization to the CWT is obvious, the precise orthonormality and completeness conditions remain to be mathematically scrutinized for each kernel.

Applications

Wavelet transforms sometimes give more useful information about a signal than the other transforms listed previously. Unlike Fourier transforms, wavelet transforms are constant Q operations, where

$$Q = f / \Delta f$$

in electronics and mechanics, with f being the frequency and Δf being the frequency resolution. Likewise for a wave distance or wavelength λ ,

$$Q = \Delta \lambda / \lambda .$$

Thus, regions of slow change are preferentially sampled at a slow rate, etc. That is, those terms dominate the wavelet transforms. This suggests that wavelet transforms might be excellent

for bandwidth reduction, which is indeed the case. Likewise, if we want a special "camera" with high resolution near the center and lower resolution farther from the center, wavelets might make good basis functions. It turns out that, to within detection noise error, the human retina is exactly describable in this manner. These illustrations hint at the great power and versatility of wavelet analysis for optimum multiresolution decomposition.

Conclusion

We hope that this very brief introduction serves to motivate the reader to study the following papers on optical wavelet transforms and to apply the concepts to their own work.



Harold H. Szu received a PhD from Rockefeller University in 1971, and has since worked at the Naval Research Laboratory. He has published more than 100 technical papers, and holds several patents. He hosted the SPIE Institute for Advanced Optical Technologies in Leesburg, Virginia, in 1986, and edited the SPIE Institute volume 634, *Optical and Hybrid Computing*, derived from that meeting. Dr. Szu is currently the information science group leader at the Naval Surface Warfare Center. His current research involves neural network pattern recognition, wavelet preprocessing, optical computing and photonics storage, and biomolecular implementations. He is a cofounder of the International Neural Network Society, has been its treasurer since its inception in 1987, and currently is its president-elect.



H. John Caulfield is a former editor of this journal and a current editorial board member of many other journals. He has published four books, well over 100 refereed journal papers, a dozen book chapters, and numerous popular articles in holography and other fields of optics. He is a Fellow of SPIE and OSA and recipient of a number of society honors.