International Conference on Space Optics—ICSO 2012

Ajaccio, Corse

9–12 October 2012

Edited by Bruno Cugny, Errico Armandillo, and Nikos Karafolas

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International Conference on Space Optics — ICSO 2012, edited by Bruno Cugny, Errico Armandillo, Nikos Karafolas
Proc. of SPIE Vol. 10564, 1056422 · © 2012 ESA and CNES · CCC code: 0277-786X/17/$18 · doi: 10.1117/12.2309043

Absolute Distance Measurement with an unraveled femtosecond frequency Comb

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Abstract—**We demonstrate the measurement of an arbitrary absolute distance with a mode-resolved frequency comb laser. By resolving the frequency comb modes individually with a virtually imaged phase array (VIPA), thousands of lasers modes are available that can be exploited for massively parallel homodyne interferometry. With this method a non-ambiguity range of 15 cm is obtained, allowing for non-incremental distance measurement in an interferometric scheme.**

***Index Terms*—Distance measurement; interferomety; femto-second frequency comb; optical metrology;**

I. INTRODUCTION

Interferometry is an accurate and powerful technique for measurement of absolute distance and displacement. For the highest accuracy, wavelength-stabilized lasers are used, which are traceable to the SI meter within a few steps. One of the disadvantages of interferometry with a single laser is the short range of non-ambiguity, which is equal to half the laser wavelength. This puts stringent requirements on the initial knowledge of the distance to be measured. To relax the limitations of single-wavelength interferometry, multi-color schemes have been developed using 2 or more lasers with different wavelengths. This increases the range of non-ambiguity at the expense of increased complexity [1]. Alternatively, a displacement can be measured, instead of an arbitrary distance, but in this case a linear guidance must be available.

Since 1983 the speed of light in vacuum has been defined to be exactly equal to c = 299 792 458 m/s. This connects the meter to the second, with the meter being defined as the distance traveled in vacuum in $1/c$ second. A powerful tool for calibration of optical frequency standards, like the wavelength-stabilized lasers, is the femtosecond frequency comb [2-6]. A time-base referenced frequency comb provides traceability to the SI second by measuring the optical frequency of the laser directly. Alternatively, a femtosecond laser can also be used as a tool for distance measurement itself. Here the pulse-train emitted by the fs laser is used as a ruler, exploiting the accurate knowledge of the distance between the pulses. Several schemes have been demonstrated based on cross-correlation measurements [7-9], spectral interferometry [10,11] and multiheterodyne interferometry using two slightly detuned frequency combs [12,13].

In this paper we provide a scheme that exploits the individual modes of the fs frequency comb for distance measurement. By resolving the output of a Michelson interferometer with a high-resolution spectrometer, homodyne interferometry with thousands of lasers is obtained [14]. This scheme connects the approaches based on multi-wavelength interferometry and spectral (dispersive) interferometry

II. MEASUREMENT SCHEME

The femtosecond comb we operate is a Ti:Sapphire oscillator with a repetition rate f_{rep} of 1 GHz and a pulse duration of 40 fs. The frequency comb is phase locked to a cesium atomic clock, providing a relative accuracy of the comb mode frequencies of 10^{-12} in 1 s. Within the spectral width of 808-828 nm about 9000 modes are present. A fraction of the light from the Ti:Sapphire laser is sent through a single mode fiber to deliver 4 mW with a clean beam profile to the Michelson interferometer. One of the arms of the interferometer can be displaced over 15 cm, which is sufficient to cover the pulse-to-pulse distance L_{pp} (30 cm). The output of the Michelson interferometer is focused on a virtually imaged phase array (VIPA), with a cylindrical lens. The VIPA provides angular dispersion of the transmitted light [15,16], with a free spectral range of 50 GHz. Therefore the VIPA output is also dispersed with a grating. Once imaged to a charge-coupled device (CCD) camera a 2 dimensional picture is obtained, revealing the individual comb modes as well-separated dots [17,18]. Here the vertical axis represents the high-resolution dispersion of the VIPA, whereas the horizontal axis shows the dispersion due to the grating. An overview of the setup is given in Fig. 1.

A tunable source (in our case an optical parametric oscillator, OPO) is used as a reference for assigning the right frequency to the right dot. Here the light from the OPO is sent through the same single mode fiber as the comb light, for perfect beam overlap. A fraction of the OPO light is simultaneously sent to a wavemeter with an accuracy of tens of megahertz. This is sufficient to distinguish the 1 GHz-spaced frequency comb modes. With the OPO several frequency markers within the optical bandwidth of the Ti:Sapphire laser are imaged onto the CCD camera. Along the vertical axis about 50 unique dots have been identified, corresponding to the 50 GHz free spectral range of the VIPA. The complete spectrum is reconstructed by stitching the dots of neighboring vertical lines, mapping the two dimension image to a calibrated frequency scale (see Fig.1).

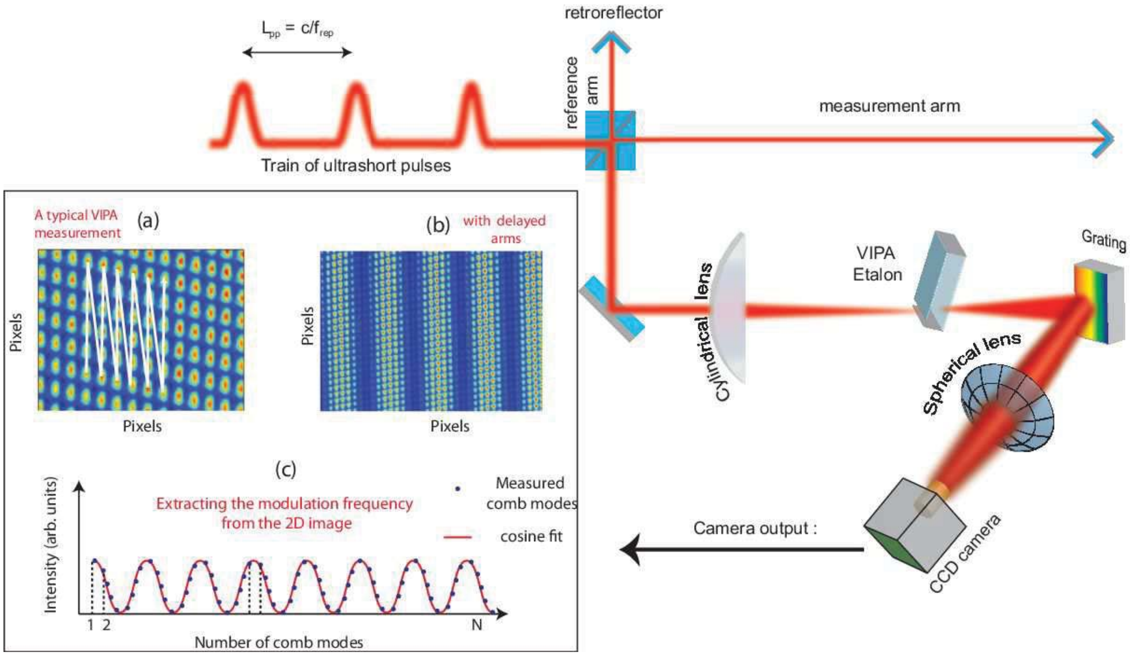

Fig. 1 Schematic overview of the setup for unraveling the output of a Michelson interferometer into distinct modes. In the inset (a) a small fraction of a typical CCD image is shown, as obtained with the measurement path blocked. Inset (b) shows a part of the CCD image when interference between the two arms occurs. The mode-resolved signal is mapped on a frequency axis by stitching together vertical lines, as schematically indicated by the white arrows (in reality one vertical line consists of about 50 dots). The result is shown in (c).

III. MEASUREMENT RESULTS

We have measured spectral interference for several path length differences between measurement and reference arm. Typical pictures are shown in Fig. 2. In Fig. 2(a) the delay between the arms is small (33 μm), leading to only a few fringes within the comb bandwidth. However, for longer delay the high resolution provided by the VIPA is essential for resolving the interference pattern, showing dark and light spots along one vertical line. Examples are shown Figs. 2(b) and 2(c), with a delay of 2.5 and 20 mm, respectively. In Fig. 2(d) the delay is set at $L_{pp}/4$ (73.9 mm), with L_{pp} the pulse-to-pulse distance $L_{pp} = c/nf_{rep}$. Here c is the speed of light in vacuum and n the refractive index of air. In this case the pulse separation is at its maximum value. In the spectral domain this leads to alternating dark and light dots; i.e., the phase difference between neighboring modes equals π. Figure 2(e) shows the pattern at 110 mm. For a distance approaching $L_{pp}/2$, consecutive pulses start overlapping and the phase difference between neighboring lines approaches 2π [Fig. 2(f)].

We use an algorithm to identify the position of the dots on the image. The power of each dot is determined by taking the integrated value of 5 × 5 pixels, largely covering an individual dot. Since the illumination of the CCD chip by the VIPA interferometer is not entirely homogeneous, these values are normalized on reference values, as obtained with one of the interferometer arms closed. As explained in the previous section, the two dimensional imaged is mapped onto a frequency scale by stitching. As an example, the unwrapped spectral interferometry data for delays of 33 μm and 2.5 mm are displayed in Fig. 3.

An absolute distance is derived from the spectral interferometry measurements by determining the phase change as a function of optical frequency. The interference term describing the output of the Michelson interferometer is proportional to cos ϕ. If dispersion is negligibly small, as in our case, the optical phase $\phi = 4\pi Lnf/c$ depends linearly on the optical frequency f and L. Here L is the one-way path length difference of the interferometer. This allows for a cosine fit through the measurement data to determine $L = cP_{mod}/(4\pi n)$, with the modulation parameter $P_{mod} = 4\pi Ln/c = \mathrm{d}\phi/\mathrm{d}f$ obtained from the curve fit. The modulation parameter is equivalent to the slope of the unwrapped phase, as obtained from methods based on fast Fourier transform [10,11]. The resolved comb frequencies are markers with a constant separation equal to the repetition rate of the laser. Therefore, the dots provide a frequency scale with a relative uncertainty of 10^{-12}. This is a huge advantage compared to lower resolution spectral interferometry, which requires careful calibration of the frequency axis [11]. Furthermore, the measurement range is not limited to a certain maximum pulse separation here. Since the comb spectrum is entirely resolved, a signal is obtained even at maximum separation of the pulses. Note that since the distance is only determined from the slope dϕ / df, the absolute optical frequency of the dots and the offset frequency f_0 have not been required so far.

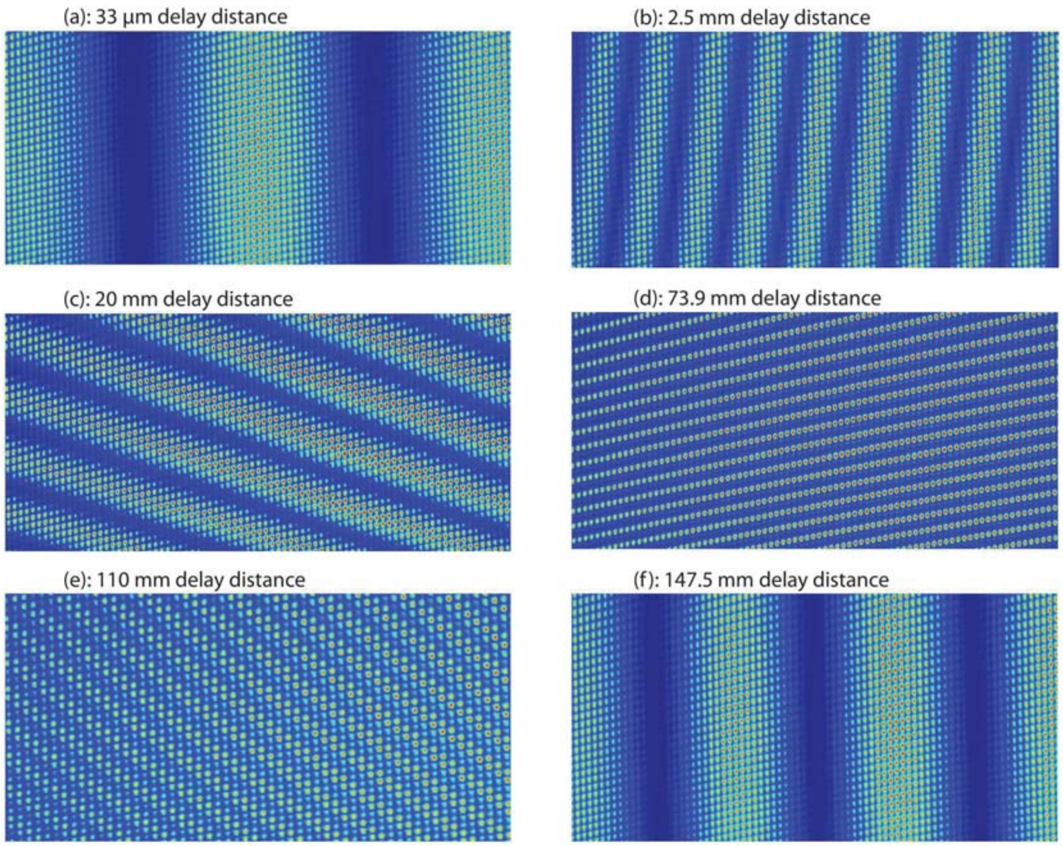

Fig. 2. Measurements obtained with the VIPA interferometer for various delays between measurement and reference path. Images (a), (b), and (c) are taken at a delay of 33 µm, 2.5 mm, and 20 mm, respectively. Image (d) shows the case of maximum pulse separation, which occurs at $L_{pp}/4$ =73.9 mm, showing π phase difference between neighboring comb modes. Images (e) and (f) are taken at a delay of 110 mm and 147.5 mm, respectively. For clarity only a quarter of the total CCD chip area is shown.

By also considering the absolute frequency of each dot, the distance measurement can be refined. Each comb line can be considered a continuous wave laser, allowing for massively parallel homodyne interferometry with thousands of lasers within the comb bandwidth. For each mode (labeled i) the distance L_i is calculated from $L_i = (m_i + \phi_i/2\pi)\lambda_i /2n_i$ for a wavelength λ_i and a corresponding refractive index n_i. The integer number of wavelengths m_i is determined from the previous measurement based on spectral interferometry. The phase for the specific wavelength λ_i, ϕ_i, is found from the cosine fit. This way of determining the phase at a certain wavelength is not sensitive to intensity variations, as is the case of homodyne interferometry with a single wavelength. For each comb wavelength a distance is determined. A final value for the distance is obtained by averaging the values as found for each comb wavelength.

IV. Comparison

We validate our measurements by comparing them to a fringe-counting wavelength-stabilized helium-neon (HeNe) laser for several distances within the scanning range of the interferometer. The HeNe laser is coupled into the measurement arm of the Michelson interferometer via a dichroic mirror, transmitting the helium-neon light at 633 nm and reflecting the comb light at 820 nm. For each position the fringe pattern is analyzed using the combined method described above. Since the HeNe laser only measures incrementally, one fringe pattern is recorded at a starting point close to zero delay, giving the one-way path length difference L_0. After displacement a second picture is recorded, providing the displacement $\Delta L = L - L_0$. The comparison between HeNe and comb measurements is plotted in Fig. 4. Averaged over all displacements measured, the difference between HeNe and comb method is about 8 nm, with a standard deviation of 28 nm (λ/30), indicating that statistical variations dominate the comparison measurement. Since the HeNe laser and the frequency comb have only the measurement path in common, limited interferometer stability and air fluctuations contribute to the difference and variation on these measurements. The measurement uncertainty of the HeNe laser is estimated to be 10–20 nanometers. The relative uncertainty on the determination of the modulation parameter (or phase), resulting from the curve fit, directly determines the relative uncertainty on the spectral interferometry measurement. This is not the case in the homodyne scheme, where the uncertainty on the phase only affects the phase fraction to be

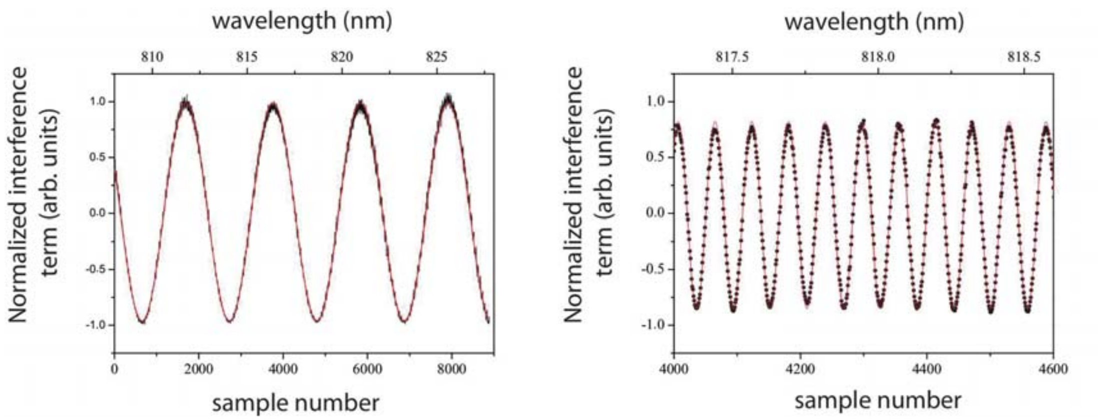

FIG. 3. The unraveled comb, with the frequency samples stitched together along the frequency axis. The frequency of a single dot is obtained from $f(p) = f_{rep}(Q - p) + f_0$, with p the sample number, Q an integer equal to 365 457, as determined by the frequency calibration, f_{rep} = 1014.82 MHz and f_0 = - 180 MHz. Left: delay of 33 µm. Right: delay of 2.5 mm, zoomed to a fraction of the full scale for clarity. The individual samples are indicated by black dots. The solid (red) curves represent a cosine fit through the data.

added to the integer m_i. The measurement uncertainty for a homodyne measurement is indicated as error bars in Fig. 4, ranging from a few to tens of nanometers, depending on the pulse separation. When measuring longer distances, e.g., hundreds of pulse-to-pulse distances, f_{rep} may be chosen such that pulse separation is small and only a few fringes are observed within the comb bandwidth. In this way the measurement uncertainty resulting from the phase determination can be minimized. As discussed above, the pulses have maximum separation at $L_{pp}/4$. In this case the fastest spectral modulation occurs, where dark and light dots alternate. In case of white light interferometry with a continuous spectrum, the spectral modulation would always increase with distance. Because of the finite number of samples (comb lines), the Nyquist theorem applies here. For distances exceeding $L_{pp}/4$ a case of undersampling occurs, resulting in decreasing spectral modulation. A slight change of f_{rep} can be used to determine whether the distance is above or below L_{pp} / 4. At a distance of L_{pp} / 2, the total path length difference equals the pulse-to-pulse difference. In this case, the phase difference between neighboring comb lines equals 2π. Here all frequency components have the same phase, but this does not occur at a coherence maximum. Since f_0 is not equal to 0, all frequency components have the phase $2\pi f_0 / f_{rep}$. For distances exceeding L_{pp} / 2, the fringe pattern starts repeating. For longer distances the integer number of L_{pp} / 2 thus needs to be known, requiring a course determination of the distance with an accuracy within the range of nonambiguity of 15 cm. This is a rather loose requirement compared to single wavelength interferometry, requiring a coarse measurement within $\lambda/2$. A measurement of the distance within 15 cm, can be obtained with, e.g., time-of-flight measurement or by changing f_{rep}.

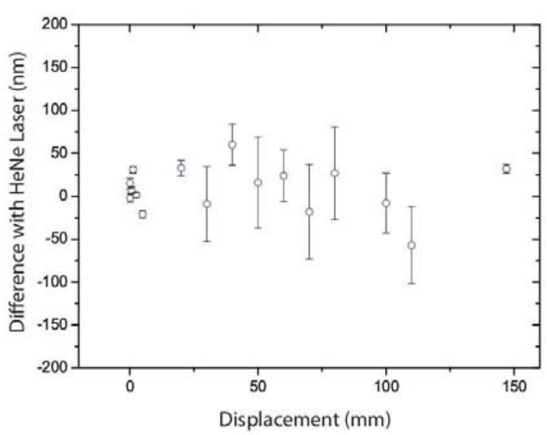

Fig. 4. Comparison between comb-based distance measurements using the VIPA spectrometer and a counting HeNe laser. The uncertainty bars represent the uncertainty on a homodyne measurement, resulting from the uncertainty on the cosine fit.

V. CONCLUSION

We have visualized interferometry with a frequency comb laser on the level of individual modes, by unraveling the output of a Michelson interferometer with a VIPA spectrometer. Interference patterns are captured within a single camera shot, containing a wealth of distance information. A distance is measured by combining spectral interferometry and homodyne interferometry with thousands of wavelengths. This results in an agreement within λ = 30, in comparison to a fringe-counting interferometer. The presented approach combines interferometry with along range of nonambiguity, allowing for nonincremental distance measurement. The measured distance can be extended to a longer range, possibly to thousands of kilometers in vacuum conditions, which may be of interest for space applications, like distance measurement between satellites. Another application may be the determination of refractive index and dispersion of materials.

ACKNOWLEDGMENT

This work is part of EURAMET joint research project "Absolute Long Distance Measurement in Air" and has received funding from the European Community's Seventh Framework Programme, ERA-NET plus, under Grant Agreement No. 217257

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