

AOA positioning algorithm based on three-dimensional space

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ABSTRACT

In order to solve the problems of large amount of calculation and a large number of false points in the case of target source in the multi sensor direction finding cross positioning system, an AOA positioning algorithm based on three-dimensional space is proposed, which introduces the sensor height information that is usually ignored in the traditional two-dimensional positioning algorithm into the algorithm. The method only needs two measurement platforms. Computer simulation results show that the algorithm has high positioning accuracy and less computation.

Keywords: Cross-positioning; AOA; False points

1. INTRODUCTION

The cross direction positioning method is one of the most widely used passive positioning methods¹⁻⁶. It uses high direction finding equipment to measure the direction of the target at two or more observation points. The intersection of each position line is the location of the target. According to the direction of the radiation source measured at each observation point and the distance of each observation point, the coordinates of the intersection point can be determined through trigonometric operation.

In the cross-positioning system with two sensors, if the information reported by one of the sensors is missing a certain one-dimensional angle information, such as elevation angle or azimuth angle, the triangulation method can be used in two-dimensional space for cross location⁷. Theoretically, the positioning accuracy of this method is worse than that of AOA positioning algorithm based on three-dimensional space, because the triangulation method only uses two information of the target, while the AOA positioning algorithm based on three-dimensional uses three information. If the GDOP estimation⁸ is performed for the cross positioning algorithm based on two-dimensional and three-dimensional space respectively, the AOA positioning algorithm based on three-dimensional space is slightly better than the triangle positioning algorithm. If the information reported by the two sensors is missing a certain dimensional angle information, such as pitch angle or azimuth angle, the three-dimensional accurate positioning of the target cannot be realized at this time, and only one directional line can be calculated. If there is other prior knowledge of the target, such as the angle of the target, etc., can be used to calculate the distance of the target, but the positioning accuracy at this time is greatly related to the accuracy of the prior knowledge⁹⁻¹¹.

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2. ALGORITHM PRINCIPLE AND PROCESS

The coordinates of the two sensors in the unified Cartesian coordinate system are respectively (X_{S1}, Y_{S1}, Z_{S1}) , (X_{S2}, Y_{S2}, Z_{S2}) . The station center polar coordinate observation of the two stations to the target is (ε_1, β_1) , (ε_2, β_2) . If there is no error, the two target angle lines should intersect at a point in space. In case of error, the two target angle rays may not intersect. Take \hat{r}_1 and \hat{r}_2 as parameters. The coordinates of the two sensors under the unified Cartesian coordinate system are respectively (X_{S1}, Y_{S1}, Z_{S1}) , (X_{S2}, Y_{S2}, Z_{S2}) . The station coordinates of the 2 stations for the target are observed as (ε_1, β_1) , (ε_2, β_2) . The process of minimum distance cross-location association algorithm is as follows. If there is no error, the two target angle lines should intersect at a point in space. In case of error, the two target angle rays may not intersect. At this time, the two closest points on the two target angle rays are used as the estimation of the target position by the two sensors. Take \hat{r}_1 and \hat{r}_2 as parameters, if the coordinate axis pointing error of the center coordinates of the two stations is ignored, two straight lines in space can be obtained. As is shown in the figure 1.

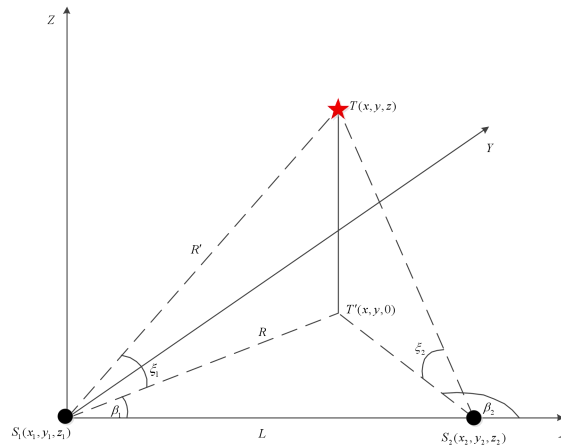


Figure 1. 2 Relation between Stations and Target

The key angle information in the figure can be represented by sentry coordinates and target coordinates as follows:

$$\left\{ \begin{array}{l} \tan \beta_1 = \frac{y - y_1}{x - x_1} \\ \tan \beta_2 = \frac{y - y_2}{x - x_2} \\ \tan \xi_1 = \frac{z - z_1}{\sqrt{(x - x_1)^2 + (y - y_1)^2}} \\ \tan \xi_2 = \frac{z - z_2}{\sqrt{(x - x_2)^2 + (y - y_2)^2}} \end{array} \right. \quad (1)$$

Where, $\tan \beta_1$ is the azimuth of the target relative to sentry one, $\tan \beta_2$ is the azimuth of the target relative to sentry two, $\tan \xi_1$ is the pitch angle of the target relative to sentry one, $\tan \xi_2$ is the pitch angle of the target relative to sentry two.

From three of these four angles, combined with the coordinates of the sentry post, the position of the target can be deduced, which is as follows:

Step1: Distance between two sentries $L = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

Step2: In $\Delta S_1 T' S_2$, from the sine theorem

$$\frac{\sin(\beta_2 - \beta_1)}{L} = \frac{\sin(\pi - \beta_2)}{R} = \frac{\sin(\beta_2)}{R} \quad (2)$$

$$R = \frac{L \sin(\beta_2)}{\sin(\beta_2 - \beta_1)} \quad (3)$$

Step3: In $\Delta T T' S_1$, distance between the target T and sentry one is

$$R' = \frac{R}{\cos \xi_1} \quad (4)$$

Step4: The coordinates of the target T can be obtained by the above formulas

$$\begin{cases} x = R' \cos \xi_1 \cos \beta_1 + x_1 \\ y = R' \cos \xi_1 \sin \beta_1 + y_1 \\ z = R' \sin \xi_1 + z_1 \end{cases} \quad (5)$$

3. FALSE POSITIONING POINT ELIMINATION ALGORITHM

If there is no error, the two target angle lines should intersect at a point in space; When there is an error, the two target angle lines may not intersect. At this time, the two closest points on the two target angle lines are used as the estimation of the target position by the two sensors respectively.

According to the geometric relationship, when the same target is correctly associated, the minimum distance between the two angle lines is small, and the fluctuation with the movement of the target is small. In false correlation, the minimum distance between two target angle lines increases obviously, and the fluctuation with the movement of the target is significant. Even if the minimum distance at a certain point is small, it will change significantly with the movement of the target. False point elimination algorithm is to use this feature to eliminate false points. The specific methods are as follows:

Calculate the minimum distance between two target angle rays according to.

The distance is judged by threshold, and if it is less than the given threshold, it is judged as the correct correlation point. If it is greater than or equal to the threshold, it is judged as a false correlation point and eliminated.

4. ERROR ANALYSIS

The performance of cross-localization algorithms is usually evaluated by geometric dilution of precision (GDOP). GDOP is the ratio of the root mean square error of the estimated position of the target to the root mean square error of the measurement, which can reflect the amplification relationship between the relative geometric position of the sensor and the target position to the measurement error. On the one hand, it shows that the relative geometric position between sensor and target can affect the estimated position of target; On the other hand, different relative geometric positions between sensors and targets will produce different positioning accuracy. Therefore, GDOP can link the relative geometric position between each sensor and target with the positioning accuracy, and can accurately measure the impact of the former on the latter¹².

Generally, there are two ways to describe the GDOP curve of positioning accuracy: the contour map in the two-dimensional plane and the three-dimensional stereoscopic curve. The former is the projection of the latter in the two-dimensional plane, and the general contour map describes the changing trend of GDOP, while the three-dimensional map visually represents the overall structure of GDOP in the whole observation area. For two-station passive detection

system, the GDOP three-dimensional solid curve contains bulges in the baseline area of the two stations and is generally similar to the saddle surface structure, which cannot be reflected in the two-dimensional contour. In engineering applications, passive radar often uses the relative error geometric dilution $\Delta GDOP(\%)$ as an index to measure the positioning accuracy.

Take the total differential of β_1, β_2, ξ_1 , and ξ_2 , you can get formulas below:

$$d\beta_1 = \frac{-(y-y_1)}{(x-x_1)^2 + (y-y_1)^2} dx + \frac{-(x-x_1)}{(x-x_1)^2 + (y-y_1)^2} dy + k_1 \quad (6)$$

$$d\beta_2 = \frac{-(y-y_2)}{(x-x_2)^2 + (y-y_2)^2} dx + \frac{-(x-x_2)}{(x-x_2)^2 + (y-y_2)^2} dy + k_2 \quad (7)$$

$$d\xi_1 = \frac{-(x-x_1)(z-z_1)}{L^2 \sqrt{(x-x_1)^2 + (y-y_1)^2}} dx + \frac{-(y-y_1)(z-z_1)}{L^2 \sqrt{(x-x_1)^2 + (y-y_1)^2}} dy + \frac{\sqrt{(x-x_1)^2 + (y-y_1)^2}}{L^2} dz + k_3 \quad (8)$$

$$d\xi_2 = \frac{-(x-x_2)(z-z_2)}{L^2 \sqrt{(x-x_2)^2 + (y-y_2)^2}} dx + \frac{-(y-y_2)(z-z_2)}{L^2 \sqrt{(x-x_2)^2 + (y-y_2)^2}} dy + \frac{\sqrt{(x-x_2)^2 + (y-y_2)^2}}{L^2} dz + k_4 \quad (9)$$

Where L is the distance from sentry post one or sentry post two to the target.

$$k_1 = \frac{(y-y_1)}{(x-x_1)^2 + (y-y_1)^2} dx_1 + \frac{(x-x_1)}{(x-x_1)^2 + (y-y_1)^2} dy_1 \quad (10)$$

$$k_2 = \frac{(y-y_2)}{(x-x_2)^2 + (y-y_2)^2} dx_2 + \frac{(x-x_2)}{(x-x_2)^2 + (y-y_2)^2} dy_2 \quad (11)$$

$$k_3 = \frac{(x-x_1)(z-z_1)}{L^2 \sqrt{(x-x_1)^2 + (y-y_1)^2}} dx_1 + \frac{(y-y_1)(z-z_1)}{L^2 \sqrt{(x-x_1)^2 + (y-y_1)^2}} dy_1 + \frac{\sqrt{(x-x_1)^2 + (y-y_1)^2}}{L^2} dz_1 \quad (12)$$

$$k_4 = \frac{(x-x_2)(z-z_2)}{L^2 \sqrt{(x-x_2)^2 + (y-y_2)^2}} dx_2 + \frac{(y-y_2)(z-z_2)}{L^2 \sqrt{(x-x_2)^2 + (y-y_2)^2}} dy_2 + \frac{\sqrt{(x-x_2)^2 + (y-y_2)^2}}{L^2} dz_2 \quad (13)$$

Write the first three of the above formulas in the form of matrix.

$$dA = Fdr + dX \tag{14}$$

Therefore, the covariance matrix can be written as

$$P_{dr} = (F^T F)^{-1} F^T (P_{dA} + P_X) F (F^T F)^{-1} \tag{15}$$

Then the geometric positioning accuracy is

$$GDOP = \sqrt{\text{tr}(P_{dr})} = \sqrt{P_{dr}(1,1) + P_{dr}(2,2) + P_{dr}(3,3)} \tag{16}$$

5. SIMULATION CALCULATION AND ANALYSIS

5.1 Single target simulation

Using the algorithm proposed in this paper, two infrared radars are used to simulate the location of a single target to verify the effectiveness of the algorithm. Assuming that the direction-finding accuracy of the two infrared radars is 0.01, the distance between the two infrared radars is 1.87km, and the distance between a single target and the centers of the two infrared radars is 1km and 3km respectively, the simulation results are shown in Figures 2 and 3.

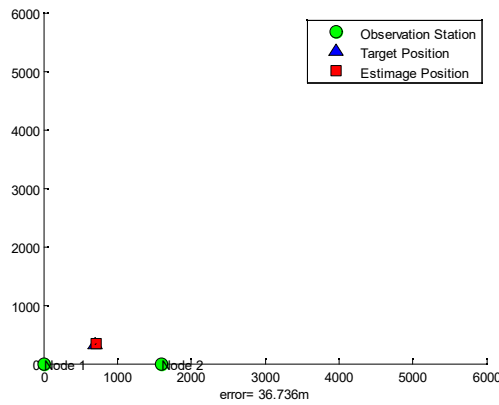


Figure 2. Single target positioning error at 1km

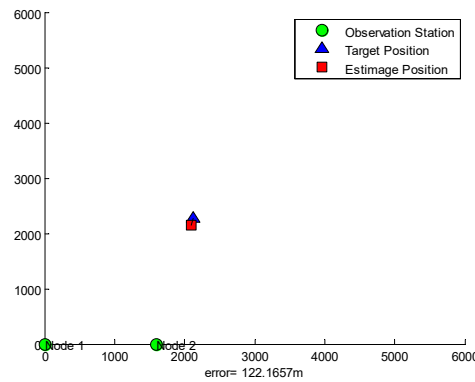


Figure 3. Single target positioning error at 3km

5.2 Multi objective simulation

Similarly, simulation experiments are carried out on the positioning of multiple targets to verify the effectiveness of the algorithm. Assuming that the direction finding accuracy of both infrared radars is 0.01, the distance between the two infrared radars is 1.87km, and the three targets are distributed at the distance of 1 km to 3 km from the centers of the two infrared radars, the simulation results are shown in Figures 4 and 5.

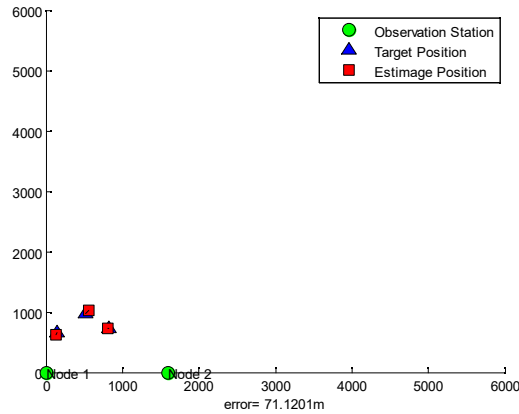


Figure 4. Multi-target positioning error at 1km

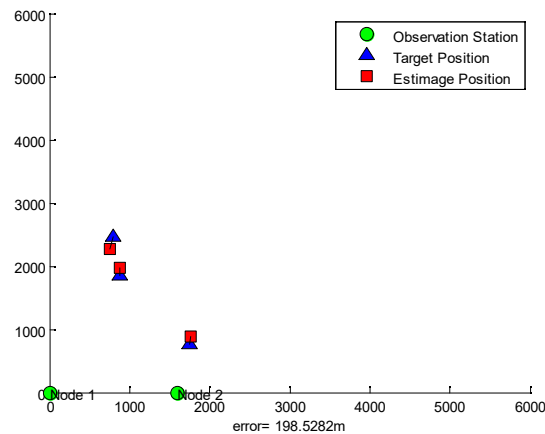


Figure 5. Multi-target positioning error at 3km

6. CONCLUSION

In this paper, the multi-target cross-localization problem in the dual infrared radar system has been researched, and a false point elimination algorithm based on minimum distance threshold is given. This method takes advantage of the fact that the minimum distance between two angle rays is small when the same target is correctly associated and the minimum distance between two angle rays is significantly increased when the same target is falsely associated. The simulation results are for a two-station system and easily extended to multi-sensor cross-positioning. At the same time, this method does not need the prior knowledge of the target, and the calculation is greatly reduced compared with the traditional method. This algorithm is mainly suitable for the situation that the sensor can accurately measure the angle

information of the target, such as the accurate tracking radar network, when the radar is interfered and can not measure the distance but can accurately measure the angle; Or an optical sensor network such as infrared.

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