Spin Hall Effect

M.I. Dyakonov

Université Montpellier II, CNRS, Montpellier, France,
dyakonov@lpta.univ-montp2.fr

Introduction

The Spin Hall Effect and related transport phenomena originating from the coupling of the charge and spin currents due to spin-orbit interaction were predicted in 1971 by Dyakonov and Perel [1, 2]. Following the suggestion in [3], the first experiments in this domain were done by Fleisher's group at Ioffe Institute in Saint Petersburg [4, 5], providing the first observation of what is now called the Inverse Spin Hall Effect. As to the Spin Hall Effect itself, it had to wait for 33 years before it was experimentally discovered by two groups in Santa Barbara (US) [6] and in Cambridge (UK) [7]. These observations aroused considerable interest and triggered intense research, both experimental and theoretical, with hundreds of publications.

The Spin Hall Effect consists in spin accumulation at the boundaries of a current-carrying conductor, the directions of the spins being opposite at the opposing boundaries. For a cylindrical wire the spins wind around the surface. The boundary spin polarization is proportional to the current and changes sign when the direction of the current is reversed.

The term "Spin Hall Effect" was introduced by Hirsch [8] in 1999. It is indeed somewhat similar to the normal Hall effect, where charges of opposite signs accumulate at the sample boundaries due to the action of the Lorentz force in magnetic field. However, there are significant differences. First, no magnetic field is needed for spin accumulation. On the contrary, if a magnetic field perpendicular to the spin direction is applied, it will destroy the spin polarization. Second, the value of the spin polarization at the boundaries is limited by spin relaxation, and the polarization exists in relatively wide *spin layers* determined by the spin diffusion length, typically on the order of 1 μm (as opposed to the much smaller Debye screening length where charges accumulate in the normal Hall effect).

Phenomenology

Spin currents. The electrons are characterized not only by charge density and electric current, but also by spin density and spin current. The spin current is described by a tensor q_{ij}, where the first index indicates the direction of flow, while the second one says which component of the spin is flowing. Thus, if all electrons with concentration n are completely spin-polarized along z and move with a velocity $\mathbf{v}$ in the x direction, the only non-zero component of q_{ij} is $q_{xz} = n\mathrm{v}$. [1]

---

1 Since s =1/2, it might be more natural to define the spin current density for this case as $(1/2)n\mathrm{v}$. It is more convenient to omit 1/2, because this allows avoiding numerous factors 1/2 and 2 in other places. It would be more correct to describe our definition of q_{ij} as the spin *polarization* current density tensor. Below we will use the

Spintronics, edited by Manijeh Razeghi, Henri-Jean M. Drouhin, Jean-Eric Wegrowe
Proc. of SPIE Vol. 7036, 70360R, (2008) · 0277-786X/08/$18 · doi: 10.1117/12.798110

Both the charge current and the spin current change sign under space inversion (because spin is a pseudo-vector). In contrast, they behave differently with respect to time inversion: while the electric current changes sign, the spin current does not (because spin changes sign under time inversion).

We will now discuss, from pure symmetry considerations, what phenomena of spin-charge coupling are in principle possible. For the moment, we restrict ourselves to an isotropic media with inversion symmetry. This does not mean that the results obtained below are not valid when inversion symmetry is absent. Rather, it means that we will not take into account additional specific effects, which are due entirely to the lack of inversion symmetry. The phenomenological approach allows to describe a number of interesting physical effects by introducing a single dimensionless parameter.

Coupling of spin and charge currents. Consider spin-up and spin-down (with respect to the z axis) electrons and suppose that we force our electrons to flow in the direction x. Let $\boldsymbol{q}\pm$ be the corresponding flow densities, which are not necessarily equal. The crucial point is that because of spin-orbit interaction these currents will induce currents of opposite signs for the two spin species in the y-direction:

$$q_y^{\pm} = \mp\gamma q_x^{\pm} , \qquad (1)$$

where γ is a dimensionless parameter proportional to the strength of the spin-orbit interaction. We assume that γ is small; the sign of γ is a priori unknown. Note, that under time inversion we have: $\boldsymbol{q}^{\pm} \rightarrow -\boldsymbol{q}^{\mp}$ Consequently, γ *changes sign* under time inversion.

We now introduce the total (charge) flow density $\boldsymbol{q} = \boldsymbol{q}^{+} + \boldsymbol{q}^{-}$ and the spin current $q_{iz} = q_i^{+} - q_i^{-}$. Mutual transformations of spin and charge currents follow from Eq. 1:

$$q_y = -\gamma q_{xz} , \quad q_{yz} = -\gamma q_x \qquad (2)$$

More accurately, the transport phenomena related to coupling of the spin and charge currents can be described phenomenologically in the following simple way [9]. We introduce the charge and spin currents, $\boldsymbol{q}^{(0)}$ and $q_{ij}^{(0)}$, which would exist in the absence of spin-orbit interaction:

$$\boldsymbol{q}^{(0)} = -\mu n\boldsymbol{E} - D\partial n/\partial \boldsymbol{r} , \qquad (3)$$

$$q_{ij}^{(0)} = -\mu E_i P_j - D\partial P_j/\partial x_i , \qquad (4)$$

where μ and D are the mobility and the diffusion coefficient, connected by the Einstein relation, n is the electron concentration, $\boldsymbol{E}$ is the electric field, and $\boldsymbol{P}$ is the vector of spin polarization density (it is convenient to use this quantity, instead of the normal spin density $\boldsymbol{S} = \boldsymbol{P}/2$, see footnote 1).

Equation 3 is the standard drift-diffusion expression for the electron flow, while Eq. 4 describes the spin current of polarized electrons, which may exist even in the absence of spin-orbit interaction, simply because spins are carried by the electron flow. We ignore possible dependence of mobility on spin polarization, which is assumed to be small. If there are other sources for currents, like for example a temperature gradient, the corresponding terms should be included in Eqs. 3 and 4.

Spin-orbit interaction couples the two currents and gives corrections to the values $\boldsymbol{q}^{(0)}$ and $q_{ij}^{(0)}$,

---

shorthand "spin current"

For an isotropic material with inversion symmetry, we have: [2]

$$q_i = q_i^{(0)} + \gamma\varepsilon_{ijk}q_{jk}^{(0)}\ , \tag{5}$$

$$q_{ij} = q_{ij}^{(0)} - \gamma\varepsilon_{ijk}q_k^{(0)}, \tag{6}$$

where q_i and q_{ij} are the corrected currents, ε_{ijk} is the unit antisymmetric tensor and γ is the small dimensionless parameter introduced above. Sums over repeating indices are assumed. The difference in signs in Eqs. 5 and 6 is due to the different properties of charge and spin currents with respect to time inversion. One can check that Eqs. 2 follow from these equations.

Phenomenological equations

Explicit phenomenological expressions for the two currents follow from Eqs. 3-6 (the electric current density $\boldsymbol{j}$ is related to $\boldsymbol{q}$ by $\boldsymbol{j} = -e\boldsymbol{q}$, where e is the elementary charge):

$$\boldsymbol{j}/e = \mu n\boldsymbol{E} + D\partial n/\partial \boldsymbol{r} + \beta\mathbf{E}\times\mathbf{P} + \delta\,\mathrm{curl}\,\boldsymbol{P}, \tag{7}$$

$$q_{ij} = -\mu E_i P_j - D\partial P_j/\partial x_i + \varepsilon_{ijk}\,(\beta n E_k + \delta\partial n/\partial x_k). \tag{8}$$

Here

$$\beta = \gamma\,\mu, \quad \delta = \gamma\,D, \tag{9}$$

so that the coefficients β and δ, similar to μ and D, satisfy the Einstein relation. However, since γ changes sign under time inversion, β and δ are *non-dissipative* kinetic coefficients, unlike μ and D.

Equations 7 and 8 should be complemented by the continuity equation for the vector of spin polarization:

$$\partial P_j/\partial t + \partial q_{ij}/\partial x_i + P_j/\tau_s = 0, \tag{10}$$

where τ_s is the spin relaxation time.

While Eqs. 7-10 are written for a three-dimensional sample, they are equally applicable to the 2D case, with obvious modifications: the electric field, space gradients, and all currents (but not the spin polarization vector) should have components in the 2D plane only.

Physical consequences of spin-charge coupling

Equations 7-10, which appeared for the first time in [1, 2] describe the physical consequences of spin-charge current coupling. The effects of spin-orbit interaction are contained in the additional terms with the coefficients β and δ.

Anomalous Hall Effect. The term $\beta\mathbf{E}\times\mathbf{P}$ in Eq. 7 describes the Anomalous Hall Effect, which is

---

2 Symmetry considerations allow for additional terms in Eq. 6, to be discussed below

observed in ferromagnets and is known for a very long time. The measured Hall voltage contains a part, which is proportional to magnetization, but cannot be explained as being due to the magnetic field produced by magnetization (it is much greater than that, especially at elevated temperatures). It took 70 years to understand [10, 11], that the Anomalous Hall Effect is due to spin-orbit interaction.

This effect can also be seen in nonmagnetic conductors, where the spin polarization is created by application of a magnetic field. The spin-related anomalous effect can be separated from the much larger ordinary Hall effect by magnetic resonance of the conduction electrons, which results in a resonant change of the Hall voltage [12]. Non-equilibrium spin polarization produced either by optical means or by spin injection, should also result in an anomalous Hall voltage. Such an experiment was recently done by Miah [13] with GaAs illuminated by circularly polarized light.

Electric current induced by* curl *P. The term δ curl $\boldsymbol{P}$ in Eq. 7 describes an electrical current induced by an inhomogeneous spin density (now referred to as the Inverse Spin Hall Effect). It can also be regarded as the diffusive counterpart of the Anomalous Hall Effect.

A way to measure this current under the conditions of optical spin orientation was proposed in [3]. The circularly polarized exciting light is absorbed in a thin layer near the surface of the sample. As a consequence, the photo-created electron spin density is inhomogeneous, however curl $\boldsymbol{P} = 0$, since $\boldsymbol{P}$ is perpendicular to the surface and it varies in the same direction. By applying a magnetic field parallel to the surface, one can create a parallel component of $\boldsymbol{P}$, thus inducing a non-zero curl $\boldsymbol{P}$ and the corresponding surface electric current (or voltage).

This effect was found by Bakun *et al* [4], providing the first experimental observation of the Inverse Spin Hall Effect, see Fig. 1. In a later publication Tkachuk *et al* [5] observed very clear manifestations of the nuclear magnetic resonance in the surface current induced by curl $\boldsymbol{P}$.

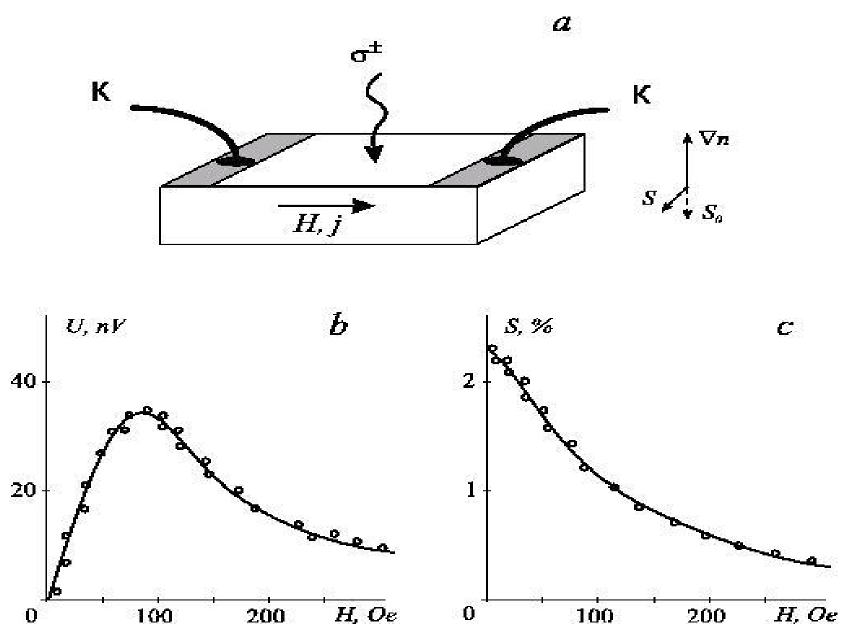

Fig. 1. First experimental observation of the Inverse Spin Hall Effect [4]. *a* - the experimental setup, *b* - voltage measured between the contacts *K* as a function of magnetic field, *c* - measured degree of circular polarization of luminescence, equal to the normal component of the average electron spin, as a function of magnetic field. The solid line in b is calculated using the results in *c*

Current-induced spin accumulation, or Spin Hall Effect. The term $\beta n \varepsilon_{ijk} E_k$ (and its diffusive counterpart $\delta \varepsilon_{ijk} \partial n/\partial x_k$) describes the Spin Hall Effect: an electrical current induces a transverse spin current, resulting in spin accumulation near the sample boundaries [1, 2]. This phenomenon was observed experimentally only in recent years [6, 7] and has attracted widespread interest.

Spin accumulation can be seen by solving Eq. 10 in the steady state ($\partial \boldsymbol{P}/\partial t = 0$) and using Eq. 8 for the spin current. Since the spin polarization will be proportional to the electric field, terms EP can be neglected. The electron concentration should be considered uniform.

We take the electric field along the x axis and look at what happens near the boundary $y = 0$ of a wide sample situated at $y > 0$ (when the sample size is greater than the spin diffusion length, spin accumulation near the other boundary can be considered independently). The boundary condition obviously should correspond to vanishing of the normal to the boundary component of the spin current, $q_{yj} = 0$.

The solution of the diffusion equation $Dd^2\boldsymbol{P}/dy^2 = \boldsymbol{P}/\tau_s$ with the boundary conditions at $y = 0$, following from Eq. 8, $dP_x/dy = 0$, $dP_y/dy = 0$, $dP_z/dy = \beta nE/D$, gives the result [1]:

$$P_z(y) = P_z(0) \exp(-y/L_s), \quad P_z(0) = -\beta nE L_s/D, \quad P_x = P_y = 0, \qquad (11)$$

where $L_s = (D\tau_s)^{1/2}$ is the spin diffusion length.

Thus the current-induced spin accumulation exists in thin layers (the *spin layers*) near the sample boundaries. The width of the spin layer is given by the spin-diffusion length, L_s, which is typically on the order of 1 μm. The polarization within the spin layer is proportional to the driving current, and the signs of spin polarization at the opposing boundaries are opposite. Fig. 2 demonstrates the first observation [6] of the Spin Hall Effect in thin films of GaAs by Kerr rotation, which has confirmed these predictions.

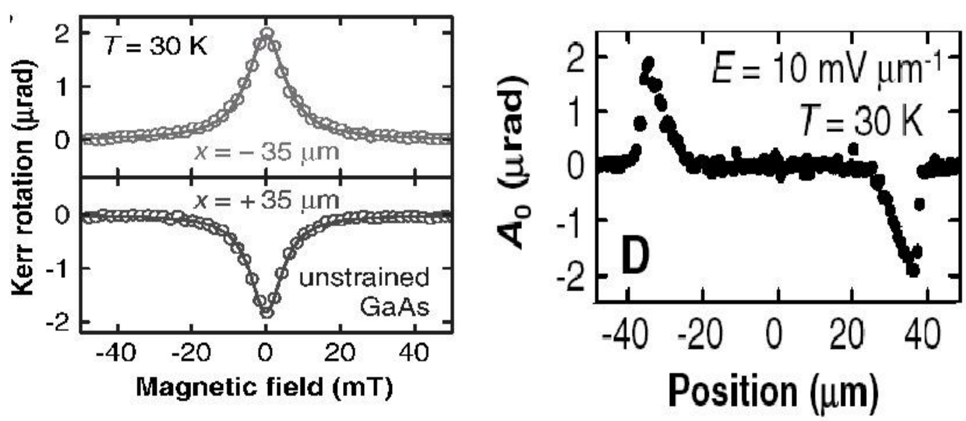

Fig. 2. First observation of the Spin Hall Effect [6]. Left panel - measurements of the Kerr rotation as a function of magnetic field at the opposing sample edges. Right panel: spatial dependence of the Kerr rotation across the channel.

It should be stressed that all these phenomena are closely related and have their common origin in the coupling between spin and charge currents given by Eqs. 5 and 6. Any mechanism that produces the Anomalous Hall Effect will also lead to the Spin Hall Effect and *vice versa*.

It is remarkable that there is a single dimensionless parameter, γ, that governs the resulting physics. The calculation of this parameter should be the objective of a microscopic theory.

The degree of polarization in the spin layer

Using Eqs. 11, 9, the *degree* of spin polarization in the spin layer, $\mathcal{P} = P_z(0)/n$, can be rewritten as

$$\mathcal{P} = \gamma(\mathrm{v}_d/\mathrm{v}_F)(3\tau_s/\tau_p)^{1/2}, \tag{12}$$

where we have introduced the electron drift velocity $\mathrm{v}_d = \mu E$ and used the conventional expression for the diffusion coefficient of degenerate 3D electrons $\mathrm{D} = \mathrm{v}_F^2\tau_p/3$, v_F is the Fermi velocity, and τ_p is the momentum relaxation time. (For 2D electrons, the factor 1/3 should be replaced by 1/2. If the electrons are not degenerate, v_F should be replaced by the thermal velocity.)

In materials with inversion symmetry, like Si, where both the spin-charge coupling and spin relaxation via the Elliott-Yaffet mechanism are due to spin asymmetry in scattering by impurities, the strength of the spin-orbit interaction cancels out in Eq. 12, since $\tau_s \sim \gamma^{-2}$.

Thus the most optimistic estimate for the degree of polarization whithin the spin layer is $\mathcal{P} \sim \mathrm{v}_d/\mathrm{v}_F$ [1]. In semiconductors, this ratio may be, in principle, on the order of 1. In the absence of inversion symmetry, usually the Dyakonov-Perel mechanism makes the spin relaxation time considerably shorter, which is unfavourable for an appreciable spin accumulation.

The validity of the approach based on the diffusion equation

The diffusion equation is valid, when the scale of spatial variation of concentration (in our case, of spin polarization density) is large compared to the mean free path $l = \mathrm{v}_F\tau_p$. The variation of P occures on the spin diffusion length, so the condition $L_s >> l$ should be satisfied. Since $L_s \sim l(\tau_s/\tau_p)^{1/2}$, this condition can be equivalently re-written as $\tau_s >> \tau_p$.

Thus, if the spin relaxation time becomes comparable to the momentum relaxation time (which is the case of the so-called "clean limit", when the spin band splitting is greater than $\hbar/\tau_p$), the diffusion equation approach breaks down. The diffusion equation still can be derived for spatial scales much greater than l, but it will be of no help for the problem at hand, because neither this equation, nor the boundary conditions for the spin current can any longer be used to study spin accumulation. Surface spin effects will occur on distances less than l from the boundaries and will crucially depend on the properties of the interfaces (e.g. flat or rough interface, etc). To understand what happens near the boundaries, one must address the quantum-mechanical problem of electrons reflecting from the boundary in the presence of electric field and spin-orbit interaction.

Swapping of spin currents: additional terms in Eq. 6

In fact, symmetry considerations allow additional terms in one of our basic equations, Eq. 6. Namely, it is possible to complement the rhs of Eq. 6 for the spin current by additional terms of the type: $q_{ji}^{(0)}$ (note the transposition of the indices i and j!) and $\delta_{ij}q_{kk}^{(0)}$ (the sum over repeating indices is assumed) with some new coefficients proportional to the spin-orbit interaction [1, 2]. This means that spin-orbit interaction will transform the spin currents, for example, turn $q_{xy}^{(0)}$ into q_{yx}, i. e. the flow of the y component of spin in the x direction will induce the flow of the x component in the y direction.

This *swapping* of spin currents should lead to new transport effects. For example, consider a ferromagnet, where spins are aligned in the z direction. An electrical current in the x direction will be accompanied by the spin current $q_{xz}^{(0)}$, which will induce the spin current q_{zx}. Now spins oriented along x will flow to the boundaries located in the z direction. This will lead to accumulation of x-oriented spins at these boundaries resulting in a slight rotation of the boundary magnetization around the y axis.

The physical origin of spin currents swapping will be discussed below.

Dissipationless spin transport?

In the current literature, and especially in articles designed for the general public, one can find numerous statements concerning the dissipationless spin currents (mostly associated with the so-called "intrinsic", or "Berry phase" mechanism of the Spin Hall Effect), which are proclaimed to provide the shining perspectives of reducing the power consumption in future spin-based computers, etc.

These statements should be taken with a big grain of salt. First, *all* spin currents by themselves, independently of their microscopic mechanism, are dissipationless. This is simply a consequence of the fact, mentioned in the introduction, that the spin current, unlike the charge current, does not change sign under time inversion. Related to this, is the property of the "spin Hall mobility", β, which is a dissipationless kinetic coefficient. Second, the existence of dissipationless spin currents does not mean that one can save energy, because the spin current is induced by the charge current, which *does* involve dissipation. The normal Hall current is dissipationless too. This does not mean that we should reduce energy consumption by building computers utilising Hall currents!

Phenomenology (without inversion symmetry)

If inversion symmetry is absent, whether in a bulk crystal, or in a two-dimensional structure, effects additional to those considered above can arise. In gyrotropic crystals a current can be induced by a *homogeneous* non-equilibrium spin density, as it was shown theoretically by Ivchenko and Pikus [14] and by Belinicher [15]. The first experimental demonstration of this effect was reported in [16]. Inversely, an electric current will generate a uniform spin polarization.

Phenomenologically, this sort of effects can be described by a second rank tensor Q_{ij}, which connects the pseudo-vector of spin polarization $\boldsymbol{P}$ with the polar vector of electric current $\boldsymbol{j}$:

$$j_i = Q_{ik}P_k\,, \quad P_i = R_{ik}j_k\,, \tag{13}$$

where the tensor R_{ik} is the inverse of Q_{ik}. Note, that the left- and right-hand sides of Eq. 13 behave similarly with respect to time inversion, which means that these equations describe *non-dissipative* phenomena. We refer the reader to [17] for a detailed description of these effects.

For a 2D electron gas with the Bychkov-Rashba splitting the tensor Q_{ik} can be constructed using the Bychkov-Rashba field $\boldsymbol{E}^{\mathrm{R}}$, a vector pointing in the growth direction z: $Q_{ij} = \varepsilon_{ijk}E_{\mathrm{k}}^{\mathrm{R}}$, so that Eq. 13 reduces simply to $j \sim \boldsymbol{E}^{\mathrm{R}}\times\boldsymbol{P}$. Concerning the spin current, in the latter case it may contain two additional terms. The first one is quadratic in $\boldsymbol{E}^{\mathrm{R}}$ and proportional to the in-plane electric field $\boldsymbol{E}$:

$$q_{ij} \sim (\boldsymbol{E}^{\mathrm{R}}\times\boldsymbol{E})_i E_j^{\mathrm{R}}.$$

If we don't care about the dependence on $\boldsymbol{E}^{\mathrm{R}}$, this term has the same symmetry properties as the previously considered $\varepsilon_{ijk}E_{\mathrm{k}}$ term, with $i,k = x,y$. The second term was first derived by Kalevich, Korenev, and Merkulov [18]. It is linear in the $\boldsymbol{E}^{\mathrm{R}}$ and proportional to the spin polarization:

$$q_{ij} \sim P_i E_j^{\mathrm{R}} - \delta_{ij}(\boldsymbol{P}\boldsymbol{E}^{\mathrm{R}}), \qquad i = x,y, \;\; j = x,y,z.$$

For this term, the non-zero components are $q_{xz} \sim P_x$, $q_{xz} \sim P_y$, $q_{xx} = q_{yy} \sim P_z$.

Thus, the most important new phenomena that may exist in the absence of inversion symmetry are the generation of both charge and spin currents by a *uniform* non-equilibrium spin polarization and the inverse effect of producing a bulk spin polarization by charge or spin current.

Microscopic mechanisms

The microscopic mechanisms responsible for the spin-charge coupling and their relative role are still not sufficiently well understood, in spite half-century theoretical efforts, and especially in recent years. The originally proposed mechanism for the Spin Hall Effect [1, 2] is related to the spin asymmetry in electron scattering due to spin-orbit interaction (the Mott effect [19, 20]), which was previously used to explain the Anomalous Hall Effect [10]. It is likely that this mechanism accounts for the existing experimental observations. Also related to scattering is the side jump mechanism proposed by Berger [21] in the contest of the Anomalous Hall Effect in ferromagnets and studied in detail by Nozières and Lewiner [22]. It is described as a spin-dependent lateral displacement of the electron wave packet during each scattering event. The role of the side jump effect is still not well understood.

Another, "intrinsic", mechanism was first considered by Karplus and Luttinger [11] and proposed recently for specific cases [23, 24], causing much excitement. It is related exclusively to the spin band splitting and does not involve spin asymmetry in scattering. [3]

Here we will qualitatively discuss spin-dependent effects in scattering from the point of view of classical mechanics, which allows achieving clarity and transparency lacking so far in the quantum-

3 The « universal spin Hall conductivity » predicted in [24] for 2D electrons with Bychkov-Rashba spin splitting, after a lively discussion was found to be actually *zero*. However this cancellation is characteristic for a linear in momentum spin band splitting. For other types of splitting the intrinsic mechanism does exist.

mechanical approach. Obviously the concept of the wave packet displacement should work best of all in the classical limit. A more detailed discussion can be found in [25].

Spin asymmetry in electron scattering. Mott has shown [19, 20], that spin-orbit interaction results in an asymmetric scattering of polarized electrons. If a polarized electron beam hits a target, it will deviate in a direction depending on the sign of polarization (similar to a spinning tennis ball in air). The Mott detectors based on this effect are used in high-energy facilities to analyse the electron spin polarization.

The scattering of electrons by a charged center is schematically depicted in Fig. 3. The most important element for us is the magnetic field ***B*** existing in the electron's moving frame and seen by the electron spin. This field is perpendicular to the plane of the electron trajectory and has opposite signs for electrons moving to the right and to the left of the charged center. The Zeeman energy of the electron spin in this field is, in fact, the spin-orbit interaction.

Simply *looking* at Fig. 3, one can make the following observations:

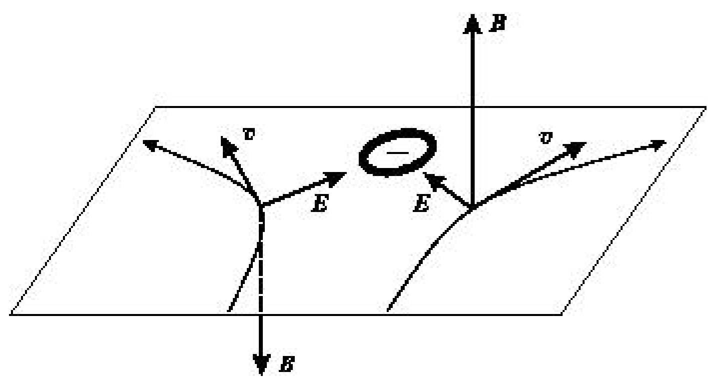

Fig. 3. Schematics of electron scattering by a negative charge. The electron spin sees a magnetic field $\boldsymbol{B} \sim \mathbf{v} \times \boldsymbol{E}$ perpendicular to the plane of the electron trajectory. Note that this magnetic field has opposite directions for electrons scattered to the left and to the right

Electron spin rotates. If the electron spin is not exactly perpendicular to the trajectory plane, it will make a precession around ***B*** during the time of collision. The angle of spin rotation during an individual collision depends on the impact parameter and on the orientation of the trajectory plane with respect to spin. This precession is at the origin of the Elliott-Yafet mechanism of spin relaxation.

The Scattering Angle Depends on Spin. The magnetic field ***B*** in Fig. 3 is inhomogeneous in space because the electric field ***E*** is non-uniform and also because the velocity **v** changes along the trajectory.

For this reason, there is a spin-dependent force (proportional to the gradient of the Zeeman energy), which acts on the electron. As a consequence, a left-right asymmetry in scattering of electrons with a given spin appears. This is the Mott effect, or *skew scattering*, resulting, among other things, in the Anomalous Hall Effect. If the incoming electrons are *not* polarized, the same spin asymmetry in scattering will result in separation of spin-up and spin-down electrons. Spin-ups will go to the right, while spin-downs will go to the left, which means that a spin current in the direction perpendicular to the incoming flux will appear (the Spin Hall Effect).

Spin rotation is correlated with scattering. As seen from Fig. 3, the spin rotation around the field $\boldsymbol{B}$ is correlated with scattering. If the spin on the right trajectory (corresponding to scattering to the right) is rotated clockwise, then the spin on the left trajectory (scattered to the left) is rotated counter-clockwise.

Let us see what happens if the incoming beam (x axis) is polarized along the y axis in the trajectory plane, i.e. characterized by a spin current q_{xy}. After scattering, the electrons going to the right will have some spin component along the x axis, while the electrons going to the left will have an x component of the opposite sign! This means that scattering transforms the initial spin current q_{xy} to q_{yx}. Similarly, q_{xx} will transform to $-q_{yy}$.

Such an analysis shows that during the scattering process the initial spin current $q_{ij}^{(0)}$ generates a new spin current q_{ij} according to the rule:

$$q_{ji}^{(0)} - \delta_{ij} q_{kk}^{(0)} \rightarrow q_{ij} .$$

Thus the correlation between spin rotation and the direction of scattering gives a physical reason for the additional terms in Eq. 6 describing the swapping of spin currents (see above). Presumably, *any* mechanism leading to the Spin Hall Effect will also contribute to such a transformation of spin currents.

The general expressions for the kinetic coefficients in Eqs. 7 through the scattering amplitude were derived in [2]. The effect of skew scattering appears only beyond the Born approximation. In contrast, the swapping of spin currents is a more robust effect: it exists already in the Born approximation.

Classical mechanics of a spinning particle. The effective mass Hamiltonian describing the spin-orbit interaction is conventionally written as

$$H_{so} = 2\lambda(\boldsymbol{k}\times \mathrm{grad} V)\boldsymbol{s} , \qquad (14)$$

where $\boldsymbol{k}$ is the electron wave vector, $\boldsymbol{s} = \boldsymbol{\sigma}/2$ is the electron spin operator, $V(\boldsymbol{r})$ is the electron potential energy, and λ is a constant defining the strength of spin-orbit interaction. In semiconductors with the band structure of GaAs, in the limit when the effective mass m is small, the Kane model gives $\lambda = \hbar^2/(4mE_g)$ for $\Delta >> E_g$ and $\lambda = (\hbar^2/(3mE_g)(\Delta/E_g)$ for $\Delta << E_g$, where Δ is the spin-orbit splitting of the valence band and E_g is the forbidden gap.

We can eliminate the Planck constant by rewriting Eq. 14 in the form

$$H_{so} = A(\boldsymbol{p}\times \mathrm{grad} V)\boldsymbol{S} , \qquad (15)$$

Here we have introduced the constant $A = 2\lambda/\hbar^2$ with the dimension (momentum)$^{-2}$ and the *dimensional* intrinsic angular momentum of the electron $\boldsymbol{S}=\hbar\boldsymbol{s}$, $\boldsymbol{p}=\hbar\boldsymbol{k}$ is the electron momentum.

We can now right down the classical Hamilton equations, corresponding to the Hamilton function $H = p^2/(2m) + V(\boldsymbol{r}) + H_{so}$:

$$d\boldsymbol{r}/dt = \boldsymbol{p}/m + A(\mathrm{grad} V\times \boldsymbol{S}), \qquad (16)$$

$$d\boldsymbol{p}/dt = -\mathrm{grad}\,[V + A(\boldsymbol{p}\times\mathrm{grad}V)\boldsymbol{S}\,], \tag{17}$$

$$d\boldsymbol{S}/dt = A(\boldsymbol{p}\times\mathrm{grad}V)\times\boldsymbol{S}. \tag{18}$$

These equations can be applied to a classical object with an internal angular momentum $\boldsymbol{S}$ (e.g. a tennis ball, with an appropriate choice of the constant A). Obviously, they are identical to the corresponding quantum-mechanical operator equations. Note, that the observable quantities are $\boldsymbol{r}$ and $\mathbf{v}=d\boldsymbol{r}/dt$, not the canonical momentum $\boldsymbol{p}$. Therefore, it may be useful to rewrite these equations in form of Newton's law for the variables $\boldsymbol{r}$ and $\mathbf{v}$. In the two-dimensional case one obtains:

$$m d\mathbf{v}/dt = -\,\mathrm{grad}V + m\,A(\mathbf{v}\times\boldsymbol{S})\,\Delta_2 V, \tag{19}$$

where Δ_2 stands for the two-dimensional Laplacian. A similar, but more complicated, equation can be easily derived for the three-dimensional case.

One can see from Eq. 19 that in the two-dimensional case the role of the spin-orbit interaction for the particle motion reduces to the action of an effective inhomogeneous magnetic field directed along $\boldsymbol{S}$ and proportional to $\Delta_2 V$.

One of the consequences of Eq. 19 is that the accelerated motion of an electron in a uniform electric field ($V=e\boldsymbol{E}\boldsymbol{r}$) is not modified by spin-orbit interaction. The *opposite statement* can be frequently found in the literature. Looking at the "anomalous velocity" (the second term in Eq. 16) one can be tempted to claim the existence of a transverse velocity $eA\boldsymbol{E}\times\boldsymbol{S}$. This is an illusion: the transverse to the electric field component of $\boldsymbol{p}$ being conserved (see Eq. 17), the transverse component of velocity is also a constant. This constant is equal to the initial value of the transverse velocity, exactly like in the absence of spin-orbit interaction. [4]

The wrong result can be obtained in the simplest way by using Eqs. 16, 17 and introducing the momentum relaxation in the Drude-like equations:

$$\begin{aligned} &d\boldsymbol{p}/dt = -e\boldsymbol{E} - \boldsymbol{p}/\tau_p \text{ (wrong!)}, \\ &\mathbf{v} = \boldsymbol{p}/m + eA\boldsymbol{E}\times\boldsymbol{S}\,. \end{aligned} \tag{20}$$

Then in the steady state one immediately obtains the famous expression $eA\boldsymbol{E}\times\boldsymbol{S}$ for the transverse to the electric field velocity. The same wrong result can be obtained by solving the Boltzmann equation, or by using more sophisticated theoretical techniques, like Keldysh formalism. The more sophisticated and non-transparent the theory is, the more retarded and advanced Green functions are involved, the more difficult it becomes to discover the mistake, which consists in the assumption implicit in Eq. (20) that it is the transverse component of the canonical momentum $\boldsymbol{p}$, which decays to zero due to collisions. In fact, it is the transverse *velocity*, which decays to zero, so that the correct

---

4 Interestingly, the final result of the side jump theory is the existence of the average transverse velocity of electrons equal to the *same* value $eA\boldsymbol{E}\times\boldsymbol{S}$, *independently of the nature of the scattering centers and their concentration*. This bizarre feature of the side-jump mechanism has caused many discussions, see for example the polemic between Smit and Berger [27-30]. We note that so far there is no regular way of calculating the transverse current due to this mechanism, which in fact relies on a correction to the Boltzmann equation (one of many corrections) on the order of d/l, where d is the scattering diameter and l is the mean free path. It can be shown [25] that for classical scattering the ratio of the side jump and the skew scattering contributions is on the order of d/l.

equation is

$$d\mathbf{v}/dt = -e\boldsymbol{E}/m - \mathbf{v}/\tau_p\,, \qquad (21)$$

which gives a zero transverse velocity in the steady state.

This is no longer true if the electric field is time-dependent (or if the spin-orbit interaction contains powers of $\boldsymbol{p}$ higher than one). For a time dependent $\boldsymbol{E}(t)$ instead of Eq. 21 we will have:

$$d\mathbf{v}/dt = -e\boldsymbol{E}/m + eA(d\boldsymbol{E}/dt \times \boldsymbol{S}) - \mathbf{v}/\tau_p\,. \qquad (22)$$

Now, for $\boldsymbol{E}\,(t)$=$\boldsymbol{E}\cos(\omega t)$ in the stationary state there will exist an oscillating transverse velocity proportional to $\omega\tau_p$.

Conclusions

The Spin Hall Effect is a new transport phenomenon, predicted a long time ago but observed only in recent years. It was experimentally studied in three- and two-dimensional semiconductors samples [6, 7, 30-33]. The Inverse Spin Hall Effect was seen in semiconductors [4, 5, 31], as well as in metals [34, 35]. Finally, it is important that these effects are observable not only at cryogenic, but also at room temperature [36]. However, the number of experimental papers is still about two orders of magnitude less than the number of theoretical ones. At present, it is difficult to predict whether this effect will have any practical applications, as many people believe, or it will belong only to fundamental research as a tool for studying spin interactions in solids.

The Spin Hall Effect shares with the long-studied Anomalous Hall Effect an uncertainty about its microscopic origin. Let us hope that future, primarily experimental, but also theoretical work will help to elucidate this problem.

Acknowledgement. I thank Maria Lifshits for helpful discussions.

References

1. M.I. Dyakonov and V.I. Perel, Pis'ma Zh. Eksp. Teor. Fiz. **13**, 657 (1971) [JETP Lett. **13**, 467 (1971)]

2. M.I. Dyakonov and V.I. Perel, Phys. Lett. A **35**, 459 (1971)

3. N.S. Averkiev and M.I. Dyakonov, Fiz. Tekh. Poluprovodn. **17**, 629 (1983) [Sov. Phys. Semicond. **17**, 393 (1983)]

4. A.A. Bakun, B.P. Zakharchenya, A.A. Rogachev, M.N. Tkachuk, and V.G. Fleisher, Pis'ma Zh. Eksp. Teor. Fiz. **40**, 464 (1984) [Sov. Phys. JETP Lett. **40**, 1293 (1984)]

5. M.N. Tkachuk, B.P. Zakharchenya, and V.G. Fleisher, Z. Eksp. Teor. Fiz. Pis'ma, **44**, 47 (1986) [Sov. Phys. JETP Lett. **44**, 59 (1986)]

6. Y.K. Kato, R.C. Myers, A.C. Gossard, and D.D. Awschalom, Science **306**, 1910 (2004)

7. J. Wunderlich, B. Kaestner, J. Sinova, and T. Jungwirth, Phys. Rev. Lett. **94**, 047204 (2005)

8. J.E. Hirsch, Phys. Rev. Lett. **83**, 1834 (1999)

9. M.I. Dyakonov, Phys. Rev. Lett. **99**, 126601 (2007)

10. J. Smit, Physica. **17**, 612 (1951); *ibid* **21**, 877 (1955); *ibid* **24**, 29 (1958)

11. R. Karplus and J.M. Luttinger, Phys. Rev. **95**, 1154 (1954)

12. J.N. Chazalviel and I. Solomon, Phys. Rev. Lett. **29**, 1676 (1972)

13. M.I. Miah, J. Phys. D: Appl. Phys. **40**, 1659 (2007)

14. E.L. Ivchenko and G.E. Pikus, Sov. Phys. JETP Lett. **27**, 604 (1978)

15. V.I. Belinicher, Phys. Lett. A **66**, 213 (1978)

16. V.M. Asnin, A.A. Bakun, A.M. Danishevskii, E.L. Ivchenko, G.E. Pikus, and A.A. Rogachev, Solid State Commun. **30**, 565 (1979)

17. S. Ganichev and E.L. Ivchenko, Spin-photogalvanics, in *Spin Physics in Semiconductors*, M.I. Dyakonov (ed). chapter 9, Springer Series in Solid-State Sciences, vol 157, Berlin (2008)

18. V.K. Kalevich, V.I. Korenev, and I.A. Merkulov, Solid State Commun. **91**, 559 (1994)

19. N.F. Mott, Proc. Roy. Soc. A **124**, 425 (1929)

20. N.F. Mott and H.S.W. Massey, *The theory of atomic collisions* (Clarendon Press, 3rd edn., 0xford, 1965)

21. L. Berger, Phys. Rev. B **2**, 4559 (1970); *ibid* **6**, 1862 (1972)

22. P. Nozières and C. Lewiner, J. Physique (Paris) **34**, 901 (1973)

23. S. Murakami, N. Nagaosa, and S.-C. Zhang, Science, **301**, 1348 (2003)

24. J. Sinova, D. Culcer, Q. Niu, N.A. Sinitsyn, T. Jungwirth, A.H. MacDonald, Phys. Rev. Lett. **92**, 126603 (2004)

25. M.I. Dyakonov and A.V. Khaetskii, Spin Hall Effect, in *Spin Physics in Semiconductors*, M.I. Dyakonov (ed), chapter 8, Springer Series in Solid-State Sciences, Berlin (2008)

26. J. Smit, Phys. Rev. B **8**, 2349 (1973)

27. L. Berger, Phys. Rev. B **8**, 2352 (1973)

28. J. Smit, Phys. Rev. B **17**, 1450 (1978)

29. L. Berger, Phys. Rev. B **17**, 1453 (1978)

30. K. Nomura, J. Wunderlich, J. Sinova, B. Kaestner, A. H. MacDonald, and T. Jungwirth, Phys. Rev. B **72**, 245330 (2005)

31. H. Zhao, E. J. Loren, H. M. van Driel, and A. L. Smirl, Phys. Rev. Lett. **96**, 246601 (2006)

32. V. Sih, V.H. Lau, R.C. Myers, V.R. Horowitz, A.C. Goassard, and D.D. Awschalom, Phys. Rev. Lett. **97**, 096605 (2006)

33. V. Sih, R.C. Myers, Y. K. Kato, W.H. Lau, A.C. Gossard, and D.D. Awschalom, Nature Physics **1**, 31 (2005)

34. S.O. Valenzuela and M. Tinkham., Nature **442**, 176 (2006)

35. T. Kimura, Y. Otani, T. Sato, S. Takahashi, and S. Maekawa, Phys. Rev. Lett. **98**, 156601 (2007)

36. N.P. Stern, S. Ghosh, G. Xiang, M. Zhu, N. Samarth, and D.D. Awschalom, Phys. Rev. Lett. **97**, 126603 (2006)