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## MATHEMATICAL MODELLING OF THE COMPLETE METROLOGY OF THE PROBA-3/ASPIICS FORMATION FLYING SOLAR CORONAGRAPH

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### I. INTRODUCTION

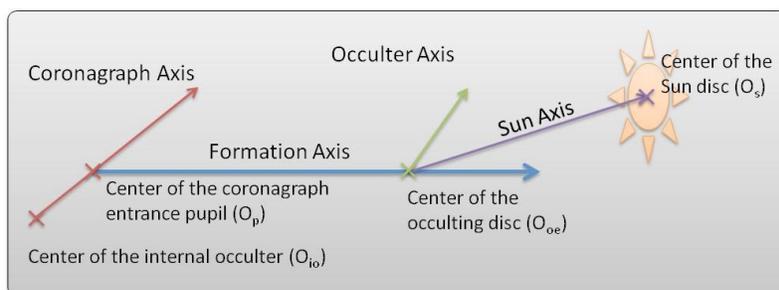
Formation flying, with ESA's mission PROBA-3, is providing the chance of creating a giant solar coronagraph in Space. The scientific payload, the solar coronagraph ASPIICS, has been selected in January 2009 [1]. The advantages of formation flying are: 1) larger dimensions for the coronagraph, which leads to better spatial resolution and lower straylight level and 2) possibility of continuous observations of the inner corona. The PROBA-3/ASPIICS mission is composed of two spacecrafts (S/Cs) at 150 meters distance, the Occulter-S/C (O-S/C) which holds the external occulter, and the Coronagraph-S/C (C-S/C) which holds the main instrument, i.e. the telescope. In addition of the scientific capabilities of the instrument, it will continuously monitor the exact position and pointing of both S/Cs in 3D space, via two additional metrology units: the Shadow Position Sensor (SPS) and the Occulter Position Sensor (OPS). In this paper we are presenting the metrology of this formation flying mission combining the outputs of the above mentioned sensors, SPS and OPS. This study has been conducted in the framework of an ESA "STARTIGER" initiative, a novel approach aimed at demonstrating the feasibility of a new and promising technology concept (in our case formation flying applied to solar coronagraphy, cf. [2, 3]) on a short time scale (six months study).

### II. DEFINITION OF THE WORKING SPACE

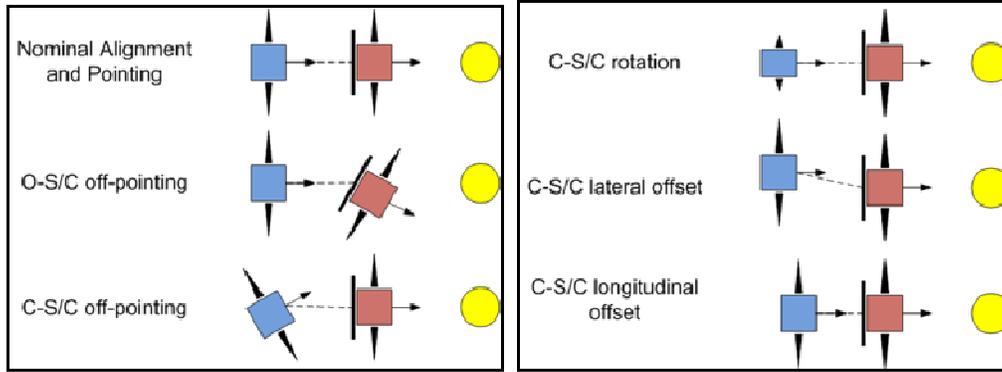
In this section we define the reference axes used to align the formation (i.e. the two spacecrafts) and to point it toward the Sun's center. Let  $O_s$  be the center of the Sun,  $O_{eo}$  that of the occulting disk,  $O_p$  that of the entrance pupil,  $O_{io}$  that of the inner occulter, and  $ISD$  (Inter-Satellite Distance) the distance between  $O_{eo}$  and  $O_p$ . As shown in Fig.1, four axes are defined:

- the optical axis of the coronagraph that is the line defined by the center of the entrance pupil  $O_p$  and the center of the inner occulter  $O_{io}$ ;
- the axis of the formation that is the line defined by the center of the entrance pupil ( $O_p$ ) and the center of the occulting disk ( $O_{eo}$ );
- the axis of the occulter that is the line perpendicular to the mean plane of the occulter and centered on the disk ( $O_{eo}$ );
- the "Sun Axis" that is the line defined by the center of the occulting disc ( $O_{eo}$ ) and the center of the Sun disc ( $O_s$ ).

Each S/C has 6 degrees of freedom (DoFs) considered as four possible displacements (lateral i.e. along x- and y-axes; longitudinal, i.e. z-axis; tilts around x- and y-axes; rotation around z-axis) as shown in Figure 2. Four displacements means 16 cases for each satellite or 256 different cases for the flying formation. The number of cases can be reduced down to 32 assuming the following two hypotheses: (i) lateral and longitudinal displacements on both S/Cs lead to similar configurations; (ii) small off-pointing of the Occulter-S/C (O-S/C) can be neglected. Then removing all similar configurations, there remain 4 individual cases on the Coronagraph S/C (C-S/C), i.e. 16 possible configurations, and only two possible configurations on the O-S/C (i.e. the nominal position and the rotation around the line of sight, z-axis).



**Fig. 1.** Definition of the different axes of the system.



**Fig. 2.** All possible displacements/misalignments of the two spacecrafts. The satellite on the left is the Coronagraph-S/C (C-S/C), while the satellite facing the Sun (circle) is the Occulter-S/C (O-S/C).

### III. METROLOGY UNITS

The two sensors we intend to use, the Shadow Positioning Sensor (SPS) and the Occulter Positioning Sensor (OPS), are going to verify continuously the alignment and the pointing of the instrument. Here is a brief description of their working principles. A detailed mathematical description of how these units are working is given in [4].

#### A. Shadow Positioning Sensor (SPS)

The SPS gives direct access to the absolute pointing of the formation and can be quantified in terms of relative positioning of the second S/C compared to the axis defined by the center of the Sun and the center of the occulting disk (hosted by the first S/C).

As shown in Fig. 3 (left), the occulting disk projects both a shadow and a penumbra which are measured by the SPS located in the C-S/C shadow/penumbra cone. From these measurements and knowing the theoretical illumination pattern of the penumbra (Fig. 3, right), it is then possible to determine the position of the C-S/C with respect to the Sun and the occulting disk (i.e. the absolute pointing of the formation).

From Fig. 3 (right), we can see the penumbra created behind the EO with a well-known illumination profile. Assuming that the occulting disk axis is aligned with the Sun axis, the penumbra illumination distribution can be distributed as circular cones pointing along the Sun axis (Fig. 4). Each cone has a unique pair of characteristic numbers which are: the angle of the cone ( $\varphi$ ), and the position of its apex ( $d$ ) on the Sun axis. From the position of the cone apex we know also the pointing of the cone. The equation of such a cone is:

$$x^2 + y^2 = \tan^2(\varphi_i) \cdot (z + d_{\varphi_i})^2 \quad (1)$$

Defining the geometrical problem, we want to find the locus of a plane (i.e. the C-S/C) inside a number of concentric cones picking up only a few points (i.e. SPS sensors) located in that plane. Substituting in (1) the dependence of the sensor position ( $x, y$ ) in terms of the roto-translation ( $x_0, y_0, z_0, \theta, \gamma$ ) we get:

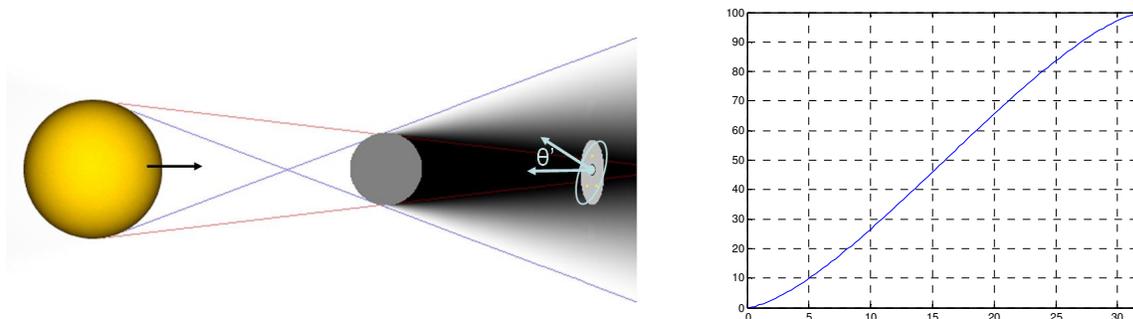
$$\begin{aligned} & (x_i \cdot \cos \gamma - y_i \cdot \sin \gamma + x_0)^2 + ((x_i \cdot \sin \gamma + y_i \cdot \cos \gamma) \cdot \cos \theta + y_0)^2 = \\ & = \tan^2(\varphi_i) \cdot ((x_i \cdot \sin \gamma + y_i \cdot \cos \gamma) \cdot \sin \theta + z_0 + d_{\varphi_i})^2 \end{aligned} \quad (2)$$

where  $(x_i, y_i)$  is the position of each SPS sensor on the C-S/C plane, and  $(\varphi_i, d_i)$  corresponds to the measurement of that sensor and the cone that it is defining. The outputs for the C-S/C are the translations  $(x_0, y_0, z_0)$ , the tilt angle  $\theta$ , and the pointing of the tilt axis  $\gamma$ .

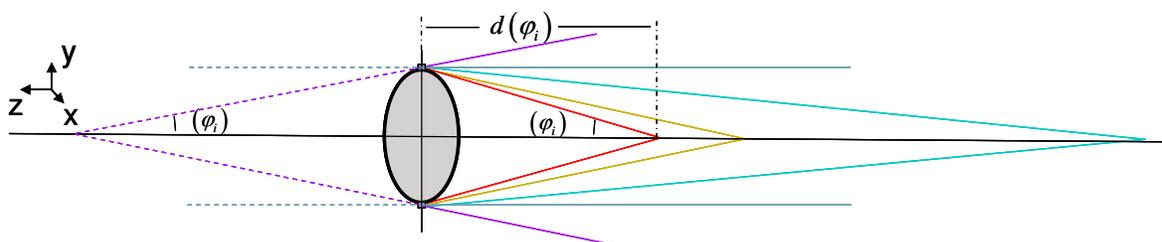
In case the occulting disk is not perpendicular to the Sun axis, the penumbra will be defined by elliptical cones (instead of circular ones),

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2} \quad (3)$$

The numerical modeling has been performed in both cases (since only the equation of the cones changes) but the difference between both cases is negligible.



**Fig. 3. LEFT:** Shadow and penumbra pattern projected behind the occulting disk. The arrows show the possible off-pointing ( $\theta'$ ) of the Coronagraph-S/C. **RIGHT:** Profile distribution of the illumination in the penumbra at 150 m beyond the occulting disk. The x-axis is the angle between the Sun-axis and the Formation Axis in arcmins; the y-axis is the % of the received Solar intensity .



**Fig. 4.** Geometrical description of the penumbra illumination distribution.

#### B. Occulter Position Sensor (OPS)

The Occulter Position Sensor (OPS) is designed to align the two satellites (i.e. the "coronagraph axis" and the "formation axis"). The OPS directly uses the scientific channel (i.e. the telescope hosted by the second S/C, the "C-S/C") to reimage light sources located on the back side of the occulting disk (at least three sources are needed, see [4] for more details). By accurately monitoring the displacement of the pattern of light sources in the field-of-view of the telescope, it is then possible to retrieve the relative positioning of the two S/Cs.

From the relative size of the 3-D reconstructed pattern with the reference pattern measured in the nominal position (this value is called similarity ratio), it is possible to retrieve longitudinal translations. When eliminating this factor, the reconstructed 3-D pattern should be the same as the initial one. From the shape of the 3-D pattern, the off-pointing of the O-S/C can easily be calculated. Finally, monitoring the decentering of the pattern with respect to the reference pattern position, the lateral translations are computed. We should note the 'lever arm' effect that is observed at the OPS when the C-S/C is off-pointed, creating additional translations which have to be subtracted (cf. Fig. 5). The equations for the final translations ( $x_0$ ,  $y_0$ ,  $z_0$ ) are:

$$x_0 = \frac{x_{dc}}{\kappa} + x_{lev}; y_0 = \frac{y_{dc}}{\kappa} - y_{lev}; z_0 = \frac{(\kappa - 1)}{\kappa} \cdot ISD + z_{lev} \quad (4)$$

where  $\kappa$  is the similarity ratio,  $(x_{dc}, y_{dc})$  is the decentering in the image,  $(x_{lev}, y_{lev}, z_{lev})$  is the 'lever arm' effect translation, and  $ISD$  is the inter-satellite distance. To conclude, a displacement of the pattern of light sources is due to either an off-pointing or a lateral displacement of the C-S/C. A shape variation of the pattern is due to either a longitudinal displacement of the C-S/C (i.e. homothetic variation) or due to an off-pointing of the O-S/C.



**Fig. 5.** Two possible cases (left: C-S/C (blue) off-pointed; right: O-S/C (red) translated) leading to similar OPS measurements.

#### IV. FINAL OUTPUTS (COMBINING SPS AND OPS)

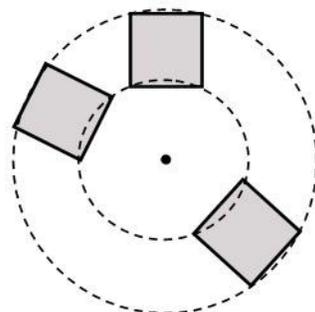
In the previous paragraph we described how the two units, SPS and OPS, are operated individually. The most challenging issue is the final combination of the SPS and OPS outputs, getting the current position and pointing of the two satellites, and determining the corrections to be applied in order to align the instrument. The SPS gives a direct access to the absolute pointing and positioning of the formation axes. From (2) we obtain the information on which S/C is translated and/or tilted. But the SPS alone is not able to distinguish the position symmetry around the Sun axis (and not around the coronagraph axis) as shown in Fig. 6. This information can be derived from the OPS assuming the O-S/C has external devices (such as star-trackers) to measure its rotation around the line of sight (i.e. Sun axis). The OPS measures the relative pointing of the coronagraph axis with respect to the axis of the formation. In other words, it will be used to quantify the positioning of the occulter in the field-of-view of the coronagraph. Thus, we can find the position of one S/C with respect to the other.

In summary, each unit gives specific information; and both have some limitations when used alone. Combining the two outputs allows to increase the capabilities of our system since more information on the formation flying and for the formation flying can be delivered. Furthermore we can validate the results as some information for S/Cs positions are common in the two outputs. What is finally missing is the orientation of the formation around the light of sight. For this information we need an external formation reference.

#### V. SCENARIOS

The number of cases can be reduced down to 32 according to the hypothesis described in section II. In reality, this can still be reduced down to 16.

Assume the O-S/C is in the correct position (i.e. the O-disk is perpendicular to the sun axis) and consider that only the C-S/C can move. This situation corresponds to 16 cases. In case of pure translation of the C-S/C, SPS and OPS results are exactly the same. In case of pure off-pointing of the C-S/C, both SPS and OPS should be able to measure it but, in fact, because of the limitation of the SPS accuracy, only the OPS will detect this misalignment. If the C-S/C is translated and off-pointed, as mentioned in the previous section, the OPS is not able to untangle translations and off-pointing. So the translation is given by the SPS alone, which is then subtracted from the OPS measurements to derive the off-pointing.



**Fig. 6.** Circular symmetry around the Sun axis. Both SPS and OPS are not able to determine the orientation of the formation around the Sun axis.

If we now consider that the O-S/C is off-pointed while the C-S/C is moved in translations and tilts (i.e. 16 other cases). In this case, theoretically, both the SPS and OPS are able to measure it. In reality, because of their limitations [2], SPS and OPS are not affected by tilts smaller than 20 arcmin.

Note that the configuration corresponding to a translation of the O-S/C and the C-S/C can be interpreted as a translation and tilt of the C-S/C only.

Finally, it exists additional cases in which the SPS is not able to give an output. This can happen in two similar configurations when SPS sensors are either in the shadow (0% Sun light brightness) or when they are outside the penumbra (100% Sun light brightness). With a limited number of sensors in the penumbra the SPS is not able to solve the calculations since, for the ellipse definition, (2), it needs five parameters/sensors. If only four or less are placed in the penumbra, we are not able to solve the system equation. The sensors outside the penumbra are "blank" since the positions they define are infinite. Even though, we know if the C-S/C is close to the Sun axis (0%), or outside the limits and close to the full exposure to Sun light (100%). In the first case we can still use the OPS and assume that the C-S/C is tilted or translated longitudinally and, thus, that the sensors are in the full shadow. In the second case the instrument should shut down and the S/Cs should be guided by other units.

## CONCLUSION

This work has been developed in the framework of the ESA STARTIGER initiative. The proposed methodology allows retrieving the position and pointing of the two spacecrafts of the formation flying via the two metrology units implemented in the ASPIICS/PROBA-3 instrument: the Occulter Position Sensor (OPS) and the Shadow Position Sensor (SPS).

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