

Analysis and applications of accommodative lenses for vision corrections

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Abstract. Analysis and applications of vision correction via accommodating intraocular lens (AIOL) are presented. By Gaussian optics, analytic formulas for the accommodation rate function (M) for two-optics and three-optics systems are derived and compared with the exact numerical results. In a single-optics AIOL, typical value of M is (0.5–1.5) D/mm, for an IOL power of (10–20) diopter. For a given IOL power, higher M is achieved in positive-IOL than negative-IOL. In the dual-optics AIOL, maximum accommodation is predicted when the front positive-optics moves toward the corneal plan and the back negative-optics moves backward. Our analytic formulas predict that greater accommodative rate may be achieved by using a positive-powered front optics, a general feature when either front or back optics is mobile. The M function is used to find the piggy-back IOL power for customized design based on the individual ocular parameters. Many of the new features demonstrated in this study can be easily realized by our analytic formulas, but not by raytracing method. © 2011 Society of Photo-Optical Instrumentation Engineers (SPIE). [DOI: 10.1117/1.3528649]

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1 Introduction

Various methods and devices have been explored for accommodating the vision of aged eyes, including accommodative intraocular lenses (AIOL)^{1,2} and surgical methods such as LASIK and laser sclera ablation.^{3,4} Raytracing method has been used to calculate the accommodation per 1.0 mm forward movement of the AIOL (the M -function) with single-optics⁵ and dual-optics.⁶ However, analytic formula for M is only available in single-optics AIOL derived from Gaussian optics.^{7,9} The study of Ho et al.⁶ in dual-optics AIOL was limited to the simple case that only one optics is mobile. Additional new features, which are not available in numerical method, became readily available in the analytic method to be presented in this study. In our earlier work¹⁰ we have presented the concept of enhanced accommodating efficiency via dual optics AIOL which, however, did not disclosed detail of the formulas.

This study will present detail of the derivation of the dual-optics analytic formulas which are compared with numerical results to show its validation in the linear regime. The roles of the corneal and AIOL power, anterior and posterior chamber depth on the accommodation rate function (M) will be explored. The complex nonlinear features of three-optics system is explicitly formulated by a geometric factor and the lens interaction. These features are difficult, if not possible, to be predicted by raytracing method. The M function will be used to find the piggy-back IOL power for customized design based on the individual ocular parameters. Finally, we will estimate the refractive errors resulted from the mis-position of the AIOL.

2 Theory

We shall first introduce the two-optics eye model consisting of the cornea and one-optics AIOL (shown in Fig. 1, left) which will be extended to a three-optics system (shown in Fig. 1, right) consisting of a dual-optics AIOL and the cornea. The three-optics system will be mathematically reduced to an effective two-optics system such that the two-optics formulas may be extended for three-optics system with certain revisions.

2.1 Two-optics System

For a relaxed (un-accommodated) eye after the insertion of an AIOL, the emmetropic state is described by an equation based on Gaussian optics^{7,8,11}

$$C = \frac{1}{1/(n/X - P) + S/n}, \quad (1)$$

where C and P are, respectively, the power of the cornea and the single-optics AIOL, which are separated by an anterior chamber depth S ; n is the aqueous refractive index. X is the posterior chamber depth and related to the axial length (L) by $L = S + X$ (see Fig. 1). In above equation, we have assumed a thin lens system with ignored thickness of the cornea and AIOL.⁷ The accommodating rate per 1.0 mm forward movement of the AIOL, defined as $M = dC/dS$ may be found by taking the derivative of the corneal power (C) with respect to the decrease of the distance (S). We found^{7,11}

$$M = (ZP)(2C + ZP)/1336 \quad (2a)$$

$$Z = 1 - SC/1336 \quad (2b)$$

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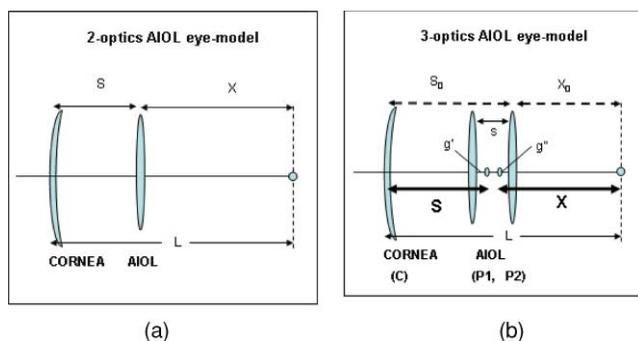


Fig. 1 Two-optics AIOL system (a) and three-optics system with a dual-optics AIOL (b).

In above equations, we have used the MKS units: X and S in mm, C and P in diopter (or $1/m$). 1336 is from the refractive index of the aqueous ($n = 1.336$) converted to the MKS units. We have also defined $dS > 0$ ($dS < 0$) for axial movement toward (backward) to the cornea. The power reduction factor (Z) has a typical value of $Z = 0.84$ for $S = 5.0$ mm, $C = 43$ diopters.

From above equations, it may be realized that the accommodation rate function (M) is proportional to the AIOL power (P). However, it should be noted that M is governed by the combined effect of the 5 ocular parameters (C, P, X, S, L) and related by the emmetropic state Eq. (1). Therefore calculating the M value will require 4 of these 5 parameters and subject to Eq. (1). Examples are show as follows: (i) for $L = 23.8$, $S = 5.0$ mm, $M = (1.1, 1.62)$ (D/mm) for and AIOL power of $P = (17, 25.6)$ D and the associate corneal power of $C = (45, 39)$ D, calculated from Eq. (1); (ii) for fixed $C = 43$ D, $L = 23.6$ mm, we found $M = (1.08, 1.36)$ (D/mm), for various $S = (2.0, 6.0)$ mm. More details will be shown later.

2.2 Three-optics System

We shall now extend the above two-optics formulas to a three-optics system as shown in Fig. 1. The dual-optics AIOL is much more complex than the single-optics AIOL due to the fact that either the front or the back optics can be mobile and the system overall power is influenced by multiple ocular parameters. In order to manipulate this complex system, we first define the AIOL effective total power (P) by its front and back optics power P_1 and P_2 ,

$$P = P_{12} - (s/1336)P_1P_2, \quad (3)$$

where $P_{12} = P_1 + P_2$; and s is the separation of the dual optics lenses. We further define the geometry factors $g' = P_2/P$ and $g'' = P_1/P$ such that S and X in Eq. (1) are revised to $X = X_0 + g''s$ and $S = S_0 - (1 - g')s$ for the case that only the front-optics is mobile; and $X = X_0 - (1 - g'')s$ and $S = S_0 + g's$ for the case that only the back-optics is mobile. These relationships may be seen by the three-optics case shown in Fig. 1.

Using Eq. (3) and the revised X and S , the derivative of Eq. (1) with respect to the lens separation (s), we obtain, after some length but straight forward derivation, the accommodation rate due to the movement of the front (M_1) and back (M_2) optics as follows

$$M_1 = gM - B, \quad (4a)$$

$$M_2 = (1 - g)M + B, \quad (4b)$$

$$g = g''[1 + 2sP_1P_2/(1336P)], \quad (4c)$$

$$B = Z^2(P_1P_2/1336). \quad (4d)$$

The above new formula for the dual-optics AIOL reduces to that of the single-optics AIOL, Eq. (2), when $s = 0$, $P_2 = 0$, $P = P_1$, $g = 1$, $B = 0$, therefore $M_2 = 0$, $M_1 = M$. The nonlinear term (B) in Eq. (4) represents the ‘‘interaction’’ in three-optics system, or the influence of the back-optics on the front-optics accommodation, and vice versa. The geometric factor (g) provides the influence of the lens separation and the power ratio $g'' = P_1/P$ on the accommodative rate (M). We note that $g < 1$ for $s > 0$, and $g = 1$, for $s = 0$. It also provides the discount factor of the accommodation rate in three-optics system comparing to the two-optics system, since $g < 1$. Equation (4c) is rigorously derived to include also the s -dependence of the g -factor, the second term of Eq. (4c).¹¹

Alternatively, Eq. (4) may be derived by the following simple argument for a deeper physics insight. In the revised X and S , the dual-optics AIOL reduced to one-optics. The net power change (P/net) due to the front optics forward-movement may be found by the power change due to both optics ($P_1 + P_2$) forward-movement minus the power change due to the backward-movement of the back-optics (P_2) and the interaction term (sB). Mathematically, above statement is given by $P/net = (P_1 + P_2)(sM) - P_2(sM) - sB$. Therefore, one may easily find M_1 given by $M_1 = (P/net)/s = g''M - B$, which is an approximate express of Eq. (4a) having an error about 2%.

2.3 Total Accommodation Amplitude (A)

Formula given by Eq. (4) is for the case that only one of the dual-optics is mobile. In general, both optics of the AIOL are allowed to move in response to the ciliary body contraction and could be in either forward or backward directions. The total accommodation amplitude (A) may be expressed by

$$A = M_1(dS_1) + M_2(dS_2), \quad (5)$$

where dS_1 and dS_2 represent, respectively, the amount of axial movement of the front and back optics, noting that dS_1 and dS_2 are positive for moving direction toward the cornea. We shall also note that Eq. (5) based on the linear summation of the accommodation amplitude of the back and front optics is a reasonable format, because we have included the interaction term (B) in Eq. (4).

2.4 Exact Numerical Solution

The analytic equations of M_j ($j = 1, 2$) shown by Eq. (4), are based on a linear theory assuming a linear accommodation rate which is true for a small movement of the optics. To study the nonlinear effects due to large movement, we calculate the corneal power changes, based on Eq. (1), for each of the 1.0 mm

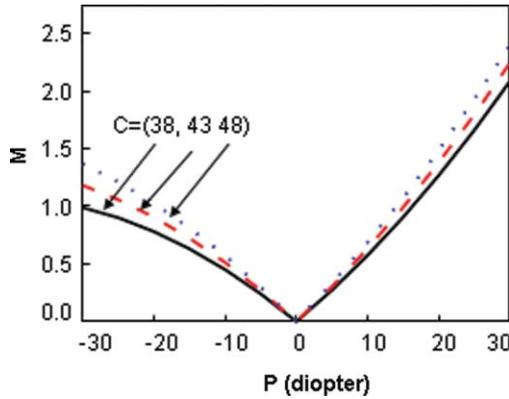


Fig. 2 Accommodation rate (M) versus IOL power (P) for various corneal powers (C).

increase of the lenses separation (s), but with different initial values of s . We shall use M_j ($j = 1,2$) = $C(s = 1.0 \text{ mm}) - C$ (at $s = 0$) for linear regime; and $M_j = C(s = 2.0 \text{ mm}) - C(s = 1.0 \text{ mm})$ for nonlinear regime. These numerical data will also justify the accuracy of our analytic formulas for the M function.

3 Results and Discussions

3.1 Single-optics AIOL

Figure 2 shows the effect of corneal power (C) on the accommodation rate (M) for various IOL power (P) from -30 to $+30$ D, where we have plot the absolute values of M . We note that higher M is found for positive-IOL (for hyperopia correction) than negative-IOL (for myopia correction). This feature may be easily realized by Eq. (2) that the $(2C + ZP)$ term has a higher value for $P > 0$ than for $P < 0$ which has a cancellation over the $2C$ term. Our analytic formula, Eq. (2), also indicates that M is a decreasing function of the anterior chamber depth (S), but is an increasing function of the product of corneal power and IOL power (PC). This implies that patient with flat cornea or lens, or short axial length is less efficient in AIOL comparing to a long axis eye or more curved cornea or lens. Also shown in Fig. 2 is the asymmetric feature of M (with respect to AIOL power signs $P > 0$ or $P < 0$) and the nonlinear behavior of M versus the IOL power (P). We should note that the above features demonstrated in Fig. 2 can be easily realized by our analytic formulas, Eq. (2), but not by raytracing method. In producing curves in Fig. 2, we have fixed $S = 3.5 \text{ mm}$ and X is found from the emmetropic state condition. Eq. (1) for a set of (P, C, S) parameters.

3.2 Dual-optics AIOL

Accommodation rate (in absolute value) for moving front-optics and moving back-optics are shown in Figs. 3 and 4, respectively, where the solid curves are the linear case based on Eq. (5) and dotted curves are the nonlinear case. We have used anterior chamber depth $S_0 = 3.5 \text{ mm}$ and lens separation $s = 2.0 \text{ mm}$ and total power of $P = 40\text{D}$. These curves justify the accuracy of our analytic formulas for M_j ($j = 1,2$). It should be noted that the linear approximation, Eq. (5), is very accurate for

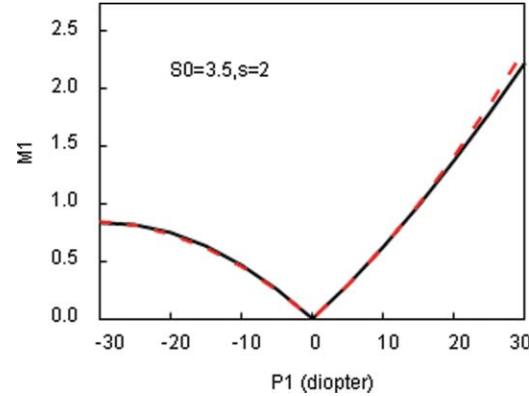


Fig. 3 Accommodation rate for mobile front-optics versus its power; solid curve is from linear Eq. (5) and dotted curve for nonlinear case.

positive-optics (with $P_1 > 0$), whereas errors occur for negative-optics (with $P_1 < 0$) particularly for high diopters. Both Figs. 3 and 4 show the asymmetric features of M versus the power signs. For front optics is mobile, M_1 is higher for $P_1 > 0$ than for $P_1 < 0$; for example, $M_1 = 1.38 \text{ (D/mm)}$ for $P = +20$ versus $M_1 = 0.83 \text{ (D/mm)}$ for $P = -20$ as shown by Fig. 3. However, an opposite trend is shown in Fig. 4 for M_2 when back-optics is mobile.

Figures 5 and 6 show the accommodation rate for mobile front and back optics, respectively, for various IOL front-optics power and for fixed anterior chamber depth $S_0 = 3.5 \text{ mm}$ and total AIOL power of $P = P_1 + P_2 = 20 \text{ D}$. We shall note that, as shown by Fig. 5, M_1 with $P_1 = +10 \text{ D}$ is slightly higher than $P_1 = -10 \text{ D}$. In addition, $M_1 = 4.5 \text{ (D/mm)}$ for $P_1 = +30 \text{ D}$ (with $P_2 = -10 \text{ D}$) with a 2.0 mm lens separation. In comparison, same M_2 value will require back optics power of $P_2 = +40 \text{ D}$ (with $P_1 = -20 \text{ D}$) when back optics is mobile, as shown in Fig. 6. M_1 is about -2 (D/mm) for the back optics (with a negative-power of $P_2 = -10 \text{ D}$, when $P_1 = +30 \text{ D}$) moving 2 mm toward the cornea. Figure 6 also shows when $P_1 = +20 \text{ D}$ (with $P_2 = 0$, for total power $P = 20 \text{ D}$), $M_2 = 0$ as expected. It should be noted that M is defined by the change rate of accommodation amplitude (A) in the forward direction, toward the cornea. Therefore positive A may be achieved for a presbyopia to see near by either a forward movement of a plus-IOL,

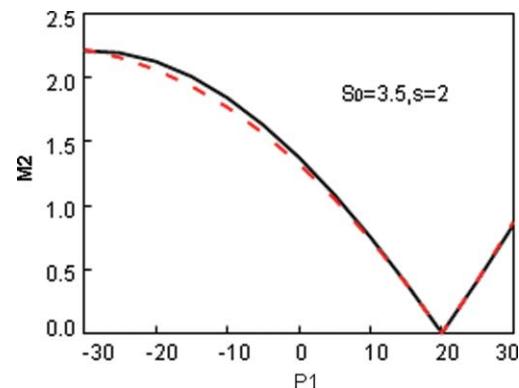


Fig. 4 Same as Fig. 3 but for mobile back-optics.

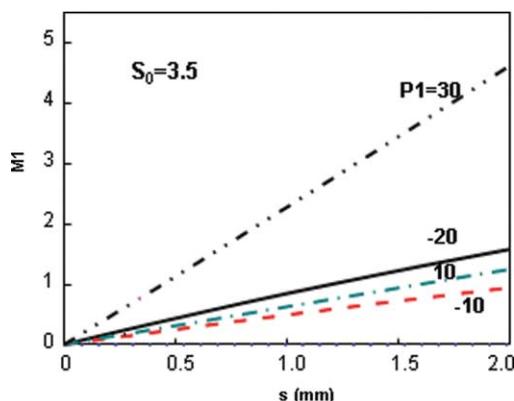


Fig. 5 Accommodation rate for moving front-optics versus the lens separation (s) for various front-optics power and for $S_0 = 3.5$ mm.

or a backward movement (toward the retina) of a minus-IOL. Greater detail for the general feature of dual-optics AIOL will be discussed in the next section.

3.3 Important Features

Many of the important features readily available from the analytic formulas of Eqs. (4) and (5) are further discussed as follows.

- When the front optics is mobile, our formulas show that higher accommodative rate (M_1) is achieved with a positive-powered front optics as opposed to a negative-optics (referred to Fig. 3). In contrast, higher M_2 is achieved with negative-powered front optics (referred to Fig. 4). Our analytic formulas readily predict that greater accommodative rate may be achieved by using a positive-powered front optics. This general feature may take a lot of trial-and-error computing time and efforts using raytracing method by Ho et al.⁶ to figure out, whereas it can be easily and precisely realized by our analytic formulas.
- For equal and same direction movement of the front and back optics with $dS_1 = dS_2 = dS$, we have $A = M(dS)$, resulted from the perfect cancelations of the gM and interaction term B in Eqs. (4a) and (4b). This also justifies that

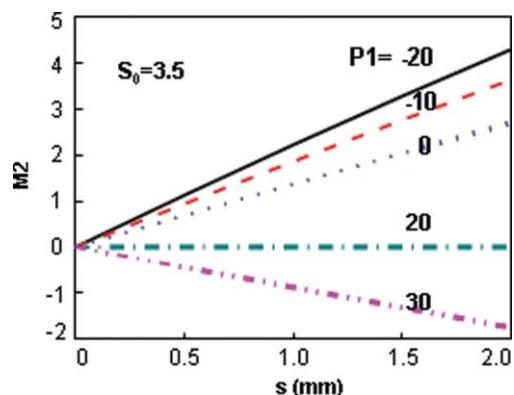


Fig. 6 Same as Figure 5 but for back optics is mobile.

our dual-optics reduced to an effective single-optics formula, Eq. (2), but having an effective total power (P) given by Eq. (3).

- For $dS_1 = dS_2$, but with opposite signs, $dS_1 > 0$, $dS_2 < 0$, we obtain $A = (2g - 1)M - 2B$, which has a maximum when $P_1 > 0$ and $P_2 < 0$ and larger than that of same direction movement in case (a). This may be easily realized by when $P_1 > 0$ and $P_2 < 0$, $B < 0$; and $(2g - 1)$ approximated by $(1 - 2P_2/P)$ has larger value for $P_2 < 0$ than for $P_2 > 0$, by an amount of $(-4P_2/P)$. The above complex nonlinear features explicitly formulated and resulted from the geometric factor (g) and the lens “interaction” term (B) in three-optics system are difficult, if not possible, to predict by raytracing.
- For maximum total accommodation amplitude (A), one would require the front positive-optics to move forward ($dS_1 > 0$) and the back negative-optics to move backward ($dS_2 < 0$) to avoid the cancellation of the first and second terms in Eq. (5). As pointed out by Ho et al.,⁶ that both optics are likely to shift and raytracing method would become very complicated. Eqs. (4) and (5) therefore provide a powerful tool for efficiency analysis since M_1 and M_2 are both analytically available.

3.4 Applications

The M function (for the single-optics AIOL) may be used to find the piggy-back IOL power (P') placed at a distance from the corneal plan $d = (S_0 - s)$ by the following relation

$$P' = P_1 - (d/Z^2)M, \quad (6)$$

where P_1 is the power of the nature-lens or IOL prior to the piggy-back placement. The revised term (d/Z^2) is a translation factor from the corneal plan to the picky-back plan.^{7,8} Equation(6) provides very useful information for surgeons in predicting the piggy-back power, because each eyes have different ocular parameters and the picky-back power must be customized accordingly.

Another application of the M function is to estimate the error caused by the uncertainty of the IOL position. It was known that it is difficult to implant an IOL at a preset position, in which the refractive error could be estimated by Eq. (1) for a set of ocular parameters. For example, for an IOL power $P = 20$ D, the refractive error caused by 1.0 mm mis-position is about 1.3 to 1.7 D depending on the corneal power of 38 to 48 D, as shown by Fig. 2, or by Eq. (2).

In conclusion, we have derived analytic formulas for the accommodation rate function (M_j) for both two-optics and three-optics systems. In the dual-optics AIOL, the total accommodation amplitude (A) has a maximum when the front positive-optics moving forward to the corneal plan and the back negative-optics moving backward. Our analytic formulas precisely predict that greater accommodative rate may be achieved by using a positive-powered front optics. This general feature is true for either front or back optics is mobile and is consistent with that of raytracing method. The complex nonlinear features of three-optics system is explicitly formulated by a geometric factor (g) and the lens “interaction” term (B). These features are difficult, if not possible, to be predicted by raytracing method. The M function may

be used to find the piggy-back IOL power (P') for customized design based on the individual ocular parameters.

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