

Binary Opening and Closing

Besides the two primary operations of erosion and dilation, there are two secondary operations that play key roles in morphological image processing, these being opening and its dual, closing. We focus mostly on opening, the properties of closing usually being analogous via complementation. Although opening is defined in terms of erosion and dilation, it possesses a more geometric formulation in terms of structuring element fits that is the basis for its application.

2.1 Opening

The **opening** of image A by image B is denoted by $A \circ B$ and is defined as a composition of erosion and dilation by

$$A \circ B = (A \ominus B) \oplus B. \quad (2.1)$$

A functional notation for opening is $\gamma_B(A)$, and the command for opening in the **MT** is `mmopen(A, B)`. For a better appreciation of the role of opening in processing, we state an equivalent formulation:

$$A \circ B = \bigcup \{B_x : B_x \subset A\}. \quad (2.2)$$

Here, the opening results from the union of all translations of the structuring element that fit inside the input image. Each fit is marked and the opening results from taking the union of the structuring element translations to each marked location. Indeed, this is precisely what is meant by eroding and then dilating.

The expression of opening as erosion followed by dilation is illustrated in Fig. 2.1, where a rectangle is eroded and then dilated by a disk. It is also possible to discern the effect of fitting, as expressed in Eq. (2.2): opening the rectangle has resulted in it being rounded from the inside, this rounding resulting from the manner in which the disk has been “rolled around” inside the rectangle to achieve a union of the fits. Had the structuring element been a small square with horizontal base, then there would have been no rounding and the opened image would have been the same as the original.

We see at once two applications of opening in Fig. 2.1. Opening by a disk results in a filter that smooths from the inside; that is, it rounds corners extending into the background. The effect is quite different with a square structuring element.

Rather than view the opened image itself as the final output of the processing, we can take a different view. We can consider the set-theoretic subtraction of the opening from the input image. This operator is called **opening top-hat**:

$$A \hat{\circ} B = A - (A \circ B). \quad (2.3)$$

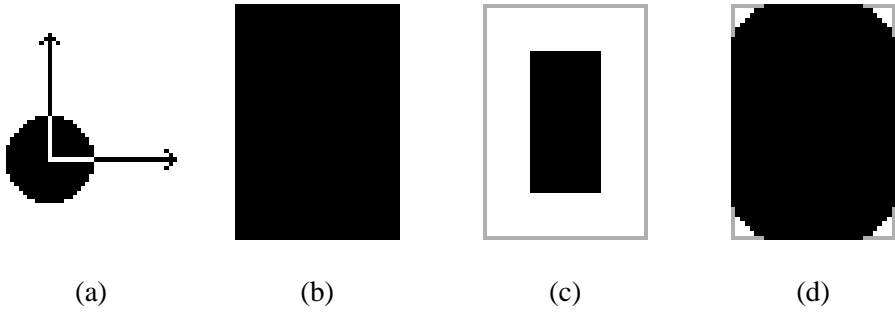


Figure 2.1 (a) Structuring element, (b) input image, (c) erosion, (d) opening.

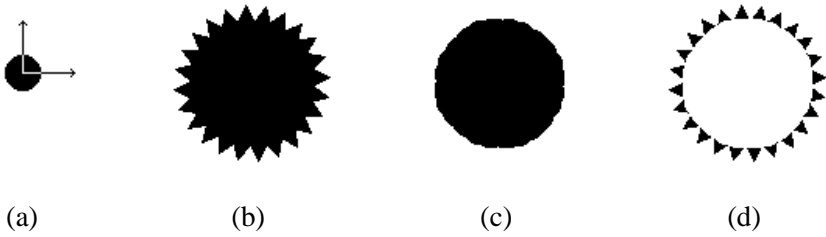


Figure 2.2 (a) Structuring element, (b) input image, (c) opening, (d) opening top-hat.

In Fig. 2.1, the opening top-hat consists of input-image corners that protrude into the background, and it can be employed for recognition purposes. Figure 2.2 shows another example of the opening top-hat to detect the teeth of a gear. Use of a disk is common because its shape-effect is rotationally invariant; however, there are many instances when it is beneficial to employ other kinds of structuring elements.

As for a digital example, if S and E are the image and structuring element of Fig. 1.9 [Eq. (1.8)], then

Example 2.1	
>>>	<code>T=mmopen(S,mmimg2se(E)) # opening of S by E</code>
>>>	<code>print T</code>
	<code>[[0 0 0 0 0 0]</code>
	<code> [0 1 0 0 0 0]</code>
	<code> [0 1 1 0 0 0]</code>
	<code> [0 1 1 1 0 0]</code>
	<code> [0 0 0 0 0 0]]</code>

If we view S as a triangle that has some background noise, then opening by E has restored the triangle. We will shortly have much more to say on this type of restoration.

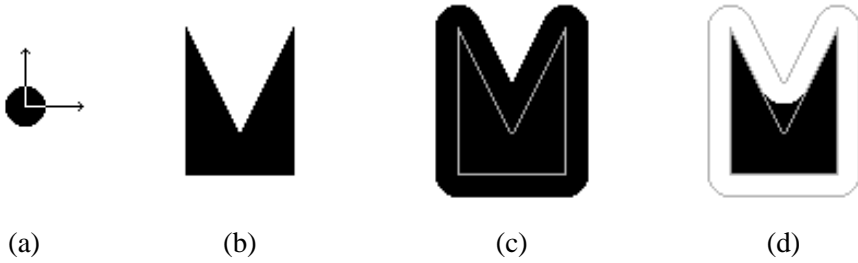


Figure 2.3 (a) Structuring element, (b) input image, (c) dilation, (d) closing.

2.2 Closing

The dual operation to opening is closing, which is defined as a dilation followed by an erosion. The **closing** of A by B is denoted by $A \bullet B$ and is defined by

$$A \bullet B = (A \oplus B) \ominus B. \quad (2.4)$$

Closing is also denoted by $\phi_B(A)$ in functional notation. In the **MT**, the close function is obtained by `mmclose(A, B)`. Figure 2.3 illustrates closing. The effect can be seen in the manner in which the closing has filtered from the outside, smoothing only corners that protrude into the image.

Closing is the dual operator of the opening because

$$A \bullet B = (A^c \circ \widetilde{B})^c. \quad (2.5)$$

Because closing is dual to opening, opening is dual to closing: replacing A by A^c in Eq. (2.5) and complementing yields

$$A \circ B = (A^c \bullet \widetilde{B})^c. \quad (2.6)$$

Note that the structuring element used in the closing is reflected. If B is a disk or any symmetrical shape, reflection plays no role. Rather than employ the iteration of Eq. (2.4) we could employ duality in conjunction with the union formulation of opening given in Eq. (2.2), thereby fitting, or “rolling the ball,” around the outside of the image. Figure 2.4 illustrates the duality between open and close with a nonsymmetrical structuring element.

As the closing contains the input image, the set-theoretical subtraction of the input image from the closing gives the **closing top-hat** operator:

$$A \hat{\bullet} B = (A \bullet B) - A. \quad (2.7)$$

Figure 2.5 shows an application of the closing and the closing top-hat. The closing of a shape by a large disk approximates its convex hull and the closing top-

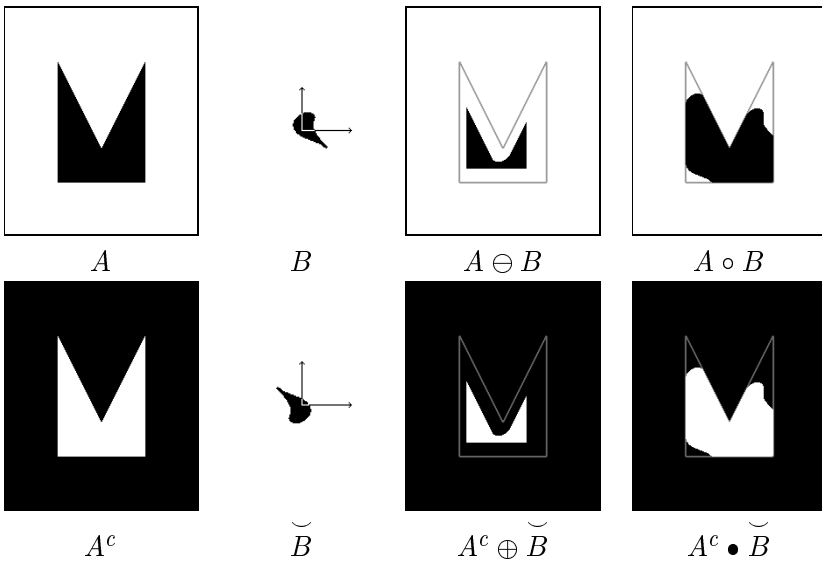


Figure 2.4 Duality between open and close with a nonsymmetrical structuring element.

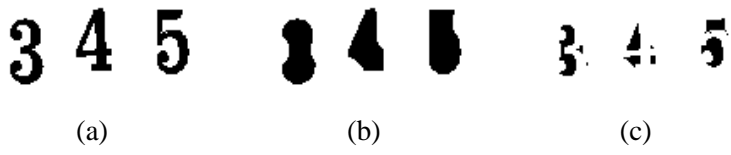


Figure 2.5 (a) Input image, (b) closing by a disk, (d) closing top-hat.

hat approximates its convex hull deficiencies. These deficiencies are commonly used in character recognition because of their powerful discriminant property.

For a digital example we close image S by structuring element E of Fig. 1.9 [Eq. (1.8)] to obtain

Example 2.2	
>>>	T = mmclose(S, mmimg2se(E)) # closing of S by E
>>>	print T
	[[0 0 0 0 0 0]
	[0 1 1 1 0 0]
	[0 1 1 1 1 0]
	[0 1 1 1 0 0]
	[0 0 0 0 0 0]]

To see the pixel effect of opening and closing, consider the image and structuring element of Fig. 1.14, the opening and closing being shown in Fig. 2.6. As

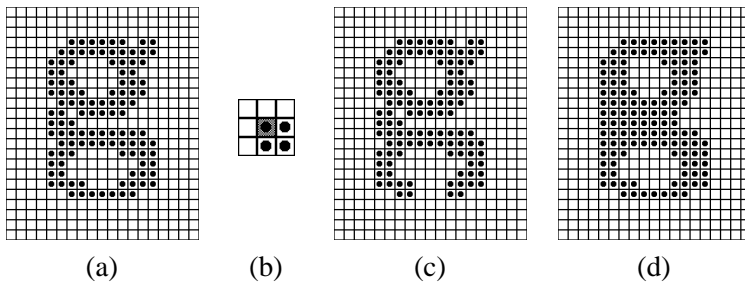


Figure 2.6 (a) Input image, (b) structuring element, (c) opening, (d) closing.

a filter, opening has cleaned the boundary by eliminating small extrusions; however, it has done this in a much finer manner than erosion, the net effect being that the opened image is a much better replica of the original than the eroded image. Analogous remarks apply to the closing, the difference being the filling of small intrusions. Note that whereas the position of the origin relative to the structuring element has a role in both erosion and dilation, it plays no role in opening and closing.

2.3 Filter Properties

Like erosion and dilation, opening and closing are both translation invariant:

$$A_x \circ B = (A \circ B)_x, \quad (2.8)$$

$$A_x \bullet B = (A \bullet B)_x. \quad (2.9)$$

They are also increasing: if A_1 is a subset of A_2 , then $A_1 \circ B$ is a subset of $A_2 \circ B$. They also satisfy some very special properties.

An operator Ψ is said to be **antiextensive** if $\Psi(A)$ is always a subset of A , and it is said to be **extensive** if $\Psi(A)$ always contains A . Opening is antiextensive: $A \circ B$ must be a subset of A . This follows from the fact that the opening is a union of translations lying within the input image. Closing is extensive: $A \bullet B$ must contain A . Therefore, the order relation

$$A \circ B \subset A \subset A \bullet B \quad (2.10)$$

is always satisfied. This property is the basis of the top-hat operators defined by Eqs. (2.3) and (2.7).

An operator Ψ is said to be **idempotent** if, for any set A , $\Psi(\Psi(A)) = \Psi(A)$. In words, operating twice by Ψ is equivalent to operating once by Ψ . Both opening and closing are idempotent:

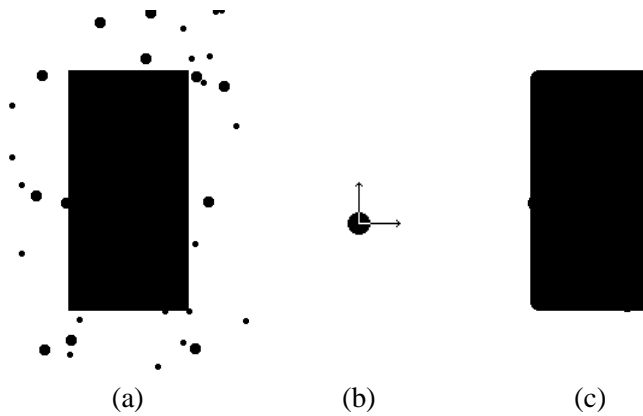


Figure 2.7 Rectangle degraded by pepper noise: (a) input image, (b) disk structuring element, (c) opening.

$$(A \circ B) \circ B = A \circ B, \quad (2.11)$$

$$(A \bullet B) \bullet B = A \bullet B. \quad (2.12)$$

The importance of idempotence is that once an image has been opened (closed), successive openings (closings) produce no further effects. This is quite different than erosion or, if we think of linear processing, moving-average filters.

In sum, whereas erosion and dilation satisfy two basic operator properties, translation invariance and increasing monotonicity, opening (closing) satisfies another two, namely antiextensivity (extensivity) and idempotence, and these play central roles in applying both opening and closing.

2.4 Application of Opening and Closing Filters

We examine the kind of restoration that can be effected by opening and closing. Consider the rectangle degraded by pepper noise in Fig. 2.7. Opening by the disk B has a restorative effect because the disk does not fit into the small components strewn about the background. Except for the rounded corners and the slight bubbles along the left and bottom edges, the rectangle has been restored. Perfect restoration could have been achieved by opening with a square; however, this would most likely be impractical, since it would require that we horizontally orient the noisy rectangle prior to filtering. Using a disk makes the filtering insensitive to rotation.

Careful consideration of the fitting formulation of opening [Eq. (2.2)] shows the manner in which the opening acts as a filter: treating the structuring element as a shape primitive, it passes only those portions of the image that are part of some translation of the shape primitive that fits inside the image. Put rigorously, a point